Fayzan Munawar
Associate *of* Royal College of Science, UK
BSc Mathematics *(Imperial College London)*
tutoring@learningmathonline.com WhatsApp: +1 718 200 2476
Facebook/Learning Math Online

Logarithmic & Exponential Functions P3

Q1

Given that $x = 4\left(3^{-y}\right)$, express y in terms of x. [3]

Q2

Solve, correct to 3 significant figures, the equation

$$e^{x} + e^{2x} = e^{3x}.$$ [5]

Q3

Solve the equation

$$\ln(x + 2) = 2 + \ln x,$$

giving your answer correct to 3 decimal places. [3]

Q4

Using the substitution $u = 3^{x}$, or otherwise, solve, correct to 3 significant figures, the equation

$$3^{x} = 2 + 3^{-x}.$$ [6]

Q5

Solve the equation $\ln\left(2 + e^{-x}\right) = 2$, giving your answer correct to 2 decimal places. [4]

Q6

Solve the equation $3^{x+2} = 3^{x} + 3^{2}$, giving your answer correct to 3 significant figures. [4]

Q7

Solve the equation

$$\ln(5 - x) = \ln 5 - \ln x,$$

giving your answers correct to 3 significant figures. [4]

Q8

Solve the equation

$$\frac{2^{x} + 1}{2^{x} - 1} = 5,$$

giving your answer correct to 3 significant figures. [4]

Q9

The variables x and y satisfy the equation $x^{n}y = C$, where n and C are constants. When $x = 1.10$, $y = 5.20$, and when $x = 3.20$, $y = 1.05$.

(i) Find the values of n and C. [5]

(ii) Explain why the graph of $\ln y$ against $\ln x$ is a straight line. [1]

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Q10

The variables x and y satisfy the equation $y^3 = Ae^{2x}$, where A is a constant. The graph of $\ln y$ against x is a straight line.

(i) Find the gradient of this line. [2]

(ii) Given that the line intersects the axis of $\ln y$ at the point where $\ln y = 0.5$, find the value of A correct to 2 decimal places. [2]

Q11

Solve the equation

$$\ln(1 + x^2) = 1 + 2\ln x,$$

giving your answer correct to 3 significant figures. [4]

Q12

The curve with equation

$$6e^{2x} + ke^y + e^{2y} = c,$$

where k and c are constants, passes through the point P with coordinates $(\ln 3, \ln 2)$.

(i) Show that $58 + 2k = c$. [2]

(ii) Given also that the gradient of the curve at P is -6, find the values of k and c. [5]

Q13

(i) Show that the equation

$$\log_2(x + 5) = 5 - \log_2 x$$

can be written as a quadratic equation in x. [3]

(ii) Hence solve the equation

$$\log_2(x + 5) = 5 - \log_2 x.$$ [2]

Q14

Use logarithms to solve the equation $5^{2x-1} = 2(3^x)$, giving your answer correct to 3 significant figures. [4]

Q15

Using the substitution $u = e^x$, or otherwise, solve the equation

$$e^x = 1 + 6e^{-x},$$

giving your answer correct to 3 significant figures. [4]

Q16

Solve the equation

$$\ln(3x + 4) = 2\ln(x + 1),$$

giving your answer correct to 3 significant figures. [4]

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Q17

Solve the equation $\ln(2x + 3) = 2\ln x + \ln 3$, giving your answer correct to 3 significant figures. [4]

Q18

Solve the equation

$$5^{x-1} = 5^x - 5,$$

giving your answer correct to 3 significant figures. [4]

Q19

Solve the equation

$$\ln(x + 5) = 1 + \ln x,$$

giving your answer in terms of e. [3]

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Answers:

Q1:

Use law for the logarithm of a product or quotient, or the logarithm of a power
Obtain $\ln x = \ln 4 - \frac{1}{3}\ln 3$, or equivalent
Obtain answer $y = \dfrac{\ln 4 - \ln x}{\ln 3}$, or equivalent

Q2:

State or imply $e^x + 1 = e^{2x}$, or $1 + e^{-x} = e^x$, or equivalent
Solve this equation as a quadratic in $u = e^x$, or in e^x, obtaining one or two roots
Obtain root $\frac{1}{2}\left(1+\sqrt{5}\right)$, or decimal in [1.61, 1.62]
Use correct method for finding x from a positive root
Obtain $x = 0.481$ and no other answer

Q3:

Use laws of logarithms and remove logarithms correctly
Obtain $x + 2 = e^2 x$, or equivalent
Obtain answer $x = 0.313$
[SR: If the logarithmic work is to base 10 then only the M mark is available.]

Q4:

State or imply at any stage that $3^{-x} = \dfrac{1}{3^x}$, or that $3^{-x} = \dfrac{1}{u}$ where $u = 3^x$
Convert given equation into the 3-term quadratic in u (or 3^x): $u^2 - 2u - 1 = 0$
Solve a 3-term quadratic, obtaining one or two roots
Obtain root $\dfrac{2+\sqrt{8}}{2}$, or a simpler equivalent, or decimal value in [2.40, 2.42]
Use a correct method for finding the value of x from a positive root
Obtain $x = 0.802$ only

Q5:

State or imply $2 + e^{-x} = e^2$
Carry out method for finding $\pm x$ from $e^{\pm x} = k$, where $k > 0$, following sound ln or exp work
Obtain $x = -\ln(e^2 - 2)$, or equivalent expression for x
Obtain answer $x = -1.68$

Q6:

Use laws of indices correctly and solve a linear equation for 3^x, or for 3^{-x}
Obtain 3^x, or 3^{-x} in any correct form, e.g. $3^x = \dfrac{3^2}{(3^2 - 1)}$
Use correct method for solving $3^{\pm x} = a$ for x, where $a > 0$
Obtain answer $x = 0.107$

Q7:

Use law of the logarithm of a product or quotient and remove logarithms
Obtain quadratic equation $x^2 - 5x + 5 = 0$, or equivalent
Solve 3-term quadratic obtaining 1 or 2 roots
Obtain answers 1.38 and 3.62

Q8:

Attempt to solve for 2^x
Obtain $2^x = 6/4$, or equivalent
Use correct method for solving an equation of the form $2^x = a$, where $a > 0$
Obtain answer $x = 0.585$

Q9:

(i) *EITHER:* State or imply $n \ln x + \ln y = \ln C$
Substitute x- and y-values and solve for n
Obtain $n = 1.50$
Solve for C
Obtain $C = 6.00$

(ii) State that the graph of $\ln y$ against $\ln x$ has equation $n \ln x + \ln y = \ln C$ which is *linear* in $\ln y$ and $\ln x$, or has equation of the form $nX + Y = \ln C$, where $X = \ln x$ and $Y = \ln y$, and is thus a straight line

Q10:

(i) State or imply $3 \ln y = \ln A + 2x$ at any stage
State gradient is $\dfrac{2}{3}$, or equivalent

(ii) Substitute $x = 0$, $\ln y = 0.5$ and solve for A
Obtain $A = 4.48$

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Q11:

Use law for the logarithm of a power, a quotient, or a product correctly at least once
Use $\ln e = 1$ or $e = \exp(1)$
Obtain a correct equation free of logarithms, e.g. $1 + x^2 = ex^2$
Solve and obtain answer $x = 0.763$ only

Q12:

(i) Use at least one of $e^{2x} = 9$, $e^y = 2$ and $e^{2y} = 4$
Obtain given result $58 + 2k = c$ **AG**

(ii) Differentiate left-hand side term by term, reaching $ae^{2x} + be^y\frac{dy}{dx} + ce^{2y}\frac{dy}{dx}$

Obtain $12e^{2x} + ke^y\frac{dy}{dx} + 2e^{2y}\frac{dy}{dx}$

Substitute $(\ln 3, \ln 2)$ in an attempt involving implicit differentiation at least once, where RHS = 0
Obtain $108 - 12k - 48 = 0$ or equivalent
Obtain $k = 5$ and $c = 68$

Use law for the logarithm of a product, power or quotient
Obtain a correct linear equation, e.g. $(2x-1)\ln 5 = \ln 2 + x\ln 3$
Solve a linear equation for x
Obtain answer $x = 1.09$

Q13:

(i) Use law for the logarithm of a product or quotient
Use $\log_2 32 = 5$ or $2^5 = 32$
Obtain $x^2 + 5x - 32 = 0$, or horizontal equivalent

(ii) Solve a 3-term quadratic equation

Obtain answer $x = 3.68$ only, or exact equivalent, e.g. $\frac{\sqrt{153}-5}{2}$

Q15:

Rearrange as $e^{2x} - e^x - 6 = 0$, or $u^2 - u - 6 = 0$, or equivalent
Solve a 3-term quadratic for e^x or for u
Obtain simplified solution $e^x = 3$ or $u = 3$
Obtain final answer $x = 1.10$ and no other

Q16:

Use law of the logarithm of a power or quotient and remove logarithms
Obtain a 3-term quadratic equation $x^2 - x - 3 = 0$, or equivalent
Solve 3-term quadratic obtaining 1 or 2 roots
Obtain answer 2.30 only

Q17:

Use law of the logarithm of a power and a product or quotient and remove logarithms
Obtain a correct equation in any form, e.g. $\frac{2x+3}{x^2} = 3$
Solve 3-term quadratic obtaining at least one root
Obtain final answer 1.39 only

Q18:

Use laws of indices correctly and solve for 5^x or for 5^{-x} or for 5^{x-1}

$$\frac{5}{1-1/5}$$

Obtain 5^x or for 5^{-x} or for 5^{x-1} in any correct form, e.g. $5^x =$
Use correct method for solving $5^x = a$, or $5^{-x} = a$, or $5^{x-1} = a$, where $a > 0$
Obtain answer $x = 1.14$

Q19:

State or imply $\ln e = 1$
Apply at least one logarithm law for product or quotient correctly (or exponential equivalent)
Obtain $x + 5 = ex$ or equivalent and hence $\frac{5}{e-1}$