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Algebra – Series Expansion P3

Q1

(i) Simplify $\left(\sqrt{(1+x)} + \sqrt{(1-x)}\right)\left(\sqrt{(1+x)} - \sqrt{(1-x)}\right)$, showing your working, and deduce that

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}} = \frac{\sqrt{(1+x)} - \sqrt{(1-x)}}{2x}.$$ [2]

(ii) Using this result, or otherwise, obtain the expansion of

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}}$$

in ascending powers of x, up to and including the term in x^2. [4]

Q2

Expand $(2 + 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

Q3

Expand $(1+x)\sqrt{(1-2x)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

Q4

When $(1+2x)(1+ax)^{\frac{2}{3}}$, where a is a constant, is expanded in ascending powers of x, the coefficient of the term in x is zero.

(i) Find the value of a. [3]

(ii) When a has this value, find the term in x^3 in the expansion of $(1+2x)(1+ax)^{\frac{2}{3}}$, simplifying the coefficient. [4]

Q5

Expand $(1 + 2x)^{-3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

Q6

Expand $\sqrt[3]{(1 - 6x)}$ in ascending powers of x up to and including the term in x^3, simplifying the coefficients. [4]

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Q7

Expand $\dfrac{16}{(2+x)^2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

Q8

(i) Expand $\dfrac{1}{\sqrt{(1-4x)}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

(ii) Hence find the coefficient of x^2 in the expansion of $\dfrac{1+2x}{\sqrt{(4-16x)}}$. [2]

Q9

Expand $\sqrt{\left(\dfrac{1-x}{1+x}\right)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

Q10

Expand $\dfrac{1}{\sqrt{(4+3x)}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

Q11

When $(1+ax)^{-2}$, where a is a positive constant, is expanded in ascending powers of x, the coefficients of x and x^3 are equal.

(i) Find the exact value of a. [4]

(ii) When a has this value, obtain the expansion up to and including the term in x^2, simplifying the coefficients. [3]

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Answers:

Q1:

(i) Simplify product and obtain $(1 + x) - (1 - x)$
  Complete the proof of the given result with no errors seen

(ii) Use correct method to obtain the first two terms of the expansion of $\sqrt{1+x}$ or $\sqrt{1-x}$

  EITHER: Obtain any correct unsimplified expansion of the numerator of the RHS of the identity

  up to the terms in x^3

  Obtain final answer with constant term $\frac{1}{3}$

  Obtain term $\frac{1}{16}x^2$ and no term in x

Q2:

: Obtain correct unsimplified version of the x or x^2 term in the expansion of $(2+3x)^{-2}$

or $(1 + \frac{3}{2}x)^{-2}$ M1

State correct first term $\frac{1}{4}$ B1

Obtain the next two terms $-\frac{3}{4}x + \frac{27}{16}x^2$ A1 + A1

[The M mark is not earned by versions with symbolic binomial coefficients such as $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.]

[The M mark is earned if division of 1 by the expansion of $(2+3x)^2$, with a correct unsimplified
  x or x^2 term, reaches a partial quotient of $a + bx$.]
[Accept exact decimal equivalents of fractions.]
[SR: Answer given as $\frac{1}{4}(1 - 3x + \frac{27}{4}x^2)$ can earn B1M1A1 (if $\frac{1}{4}$ seen but then omitted, give M1A1).]
[SR: Solutions involving $k(1 + \frac{3}{2}x)^{-2}$, where $k = 2, 4$ or $\frac{1}{2}$, can earn M1and A1√ for correctly
  simplifying both the terms in x and x^2.]

Q3:

State correct unsimplified first two terms of the expansion of $\sqrt{1-2x}$, e.g. $1 + \frac{1}{2}(-2x)$

State correct unsimplified term in x^2, e.g. $\frac{1}{2} \cdot (\frac{1}{2}-1) \cdot (-2x)^2 / 2!$

Obtain sufficient terms of the product of $(1 + x)$ and the expansion up to the term in x^2
of $\sqrt{1-2x}$

Obtain final answer $1 - \frac{3}{2}x^2$

[The B marks are not earned by versions with symbolic binomial coefficients such as $\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$.]

[SR: An attempt to rewrite $(1+x)\sqrt{1-2x}$ as $\sqrt{1-3x^2}$ earns M1 A1 and the subsequent
expansion $1 - \frac{3}{2}x^2$ gets M1 A1.]

Q4:

(i) State correct first two terms of the expansion of $(1+ax)^{\frac{2}{3}}$, i.e. $1 + \frac{2}{3}ax$ B1

  Form an expression for the coefficient of x in the expansion of $(1+2x)(1+ax)^{\frac{2}{3}}$
  and equate it to zero   M1
  Obtain $a = -3$ A1

(ii) Obtain correct unsimplified terms in x^2 and x^3 in the expansion of $(1-3x)^{\frac{2}{3}}$

  or $(1+ax)^{\frac{2}{3}}$ B1√ + B1√
  Carry out multiplication by $1 + 2x$ obtaining two terms in x^3 M1
  Obtain final answer $-\frac{10}{3}x^3$, or equivalent   A1

[Symbolic binomial coefficients, e.g. $\begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix}$, are not acceptable for the B marks in (i) or (ii)]

Q5:

Obtain $1 - 6x$

State correct unsimplified x^2 term. Binomial coefficients must be expanded.

Obtain $... + 24x^2$

Q6:

Obtain $1 + \frac{1}{3}kx$, where $k = \pm 6$ or ± 1

Obtain $1 - 2x$

Obtain $-4x^2$

Obtain $-\frac{40}{3}x^3$ or equivalent

Q7:

Obtain correct unsimplified version of x or x^2 term in expansion of

$(2 + x)^{-2}$ or $(1 + \frac{1}{2}x)^{-2}$

Correct first term 4 from correct work

Obtain $-4x$

Obtain $+ 3x^2$

Q8:

(i) Either Obtain correct (unsimplified) version of x or x^2 term from $(1-4x)^{\frac{1}{2}}$
            Obtain $1 + 2x$
            Obtain $+ 6x^2$

(ii) Combine both x^2 terms from product of $1 + 2x$ and answer from part (i)
     Obtain 5

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Q9:

State a correct unsimplified term in x or x^2 of $(1-x)^{\frac{1}{2}}$ or $(1+x)^{-\frac{1}{2}}$

State correct unsimplified expansion of $(1-x)^{\frac{1}{2}}$ up to the term in x^2

State correct unsimplified expansion of $(1+x)^{-\frac{1}{2}}$ up to the term in x^2

Obtain sufficient terms of the product of the expansions of $(1-x)^{\frac{1}{2}}$ and $(1+x)^{-\frac{1}{2}}$

Obtain final answer $1-x+\frac{1}{2}x^2$

Q10:

Obtain a correct unsimplified version of the x or x^2 term of the expansion of

$(4+3x)^{-\frac{1}{2}}$ or $(1+\frac{3}{4}x)^{-\frac{1}{2}}$

State correct first term $\dfrac{1}{2}$

Obtain the next two terms $-\dfrac{3}{16}x+\dfrac{27}{256}x^2$ A1

Q11:

(i) Obtain correct unsimplified terms in x and x^3 I

Equate coefficients and solve for a

Obtain final answer $a=\dfrac{1}{\sqrt{2}}$, or exact equivalent

(ii) Use correct method and value of a to find the first two terms of the expansion $(1+ax)^{-2}$

Obtain $1-\sqrt{2}x$, or equivalent

Obtain term $\dfrac{3}{2}x^2$

[Symbolic coefficients, e.g. a, are not sufficient for the first B marks]

[The f.t. is solely on the value of a.]