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Quadratic Functions P1

Q1

Find the value of the constant c for which the line $y = 2x + c$ is a tangent to the curve $y^2 = 4x$. [4]

Q2

Find the real roots of the equation $\frac{18}{x^4} + \frac{1}{x^2} = 4$. [4]

Q3

The function f is defined by $f(x) = a + b\cos 2x$, for $0 \leqslant x \leqslant \pi$. It is given that $f(0) = -1$ and $f(\frac{1}{2}\pi) = 7$.

(i) Find the values of a and b. [3]

(ii) Find the x-coordinates of the points where the curve $y = f(x)$ intersects the x-axis. [3]

(iii) Sketch the graph of $y = f(x)$. [2]

Q4

Determine the set of values of the constant k for which the line $y = 4x + k$ does not intersect the curve $y = x^2$. [3]

Q5

The equation of a curve is $y = x^2 - 3x + 4$.

(i) Show that the whole of the curve lies above the x-axis. [3]

(ii) Find the set of values of x for which $x^2 - 3x + 4$ is a decreasing function of x. [1]

The equation of a line is $y + 2x = k$, where k is a constant.

(iii) In the case where $k = 6$, find the coordinates of the points of intersection of the line and the curve. [3]

(iv) Find the value of k for which the line is a tangent to the curve. [3]

Q6

Find the set of values of k for which the line $y = kx - 4$ intersects the curve $y = x^2 - 2x$ at two distinct points. [4]

Q7

Determine the set of values of k for which the line $2y = x + k$ does not intersect the curve $y = x^2 - 4x + 7$. [4]

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Q8

The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x - b)^2 - c$. [3]

Q9

The function $f : x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = f(x)$. [4]

(ii) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

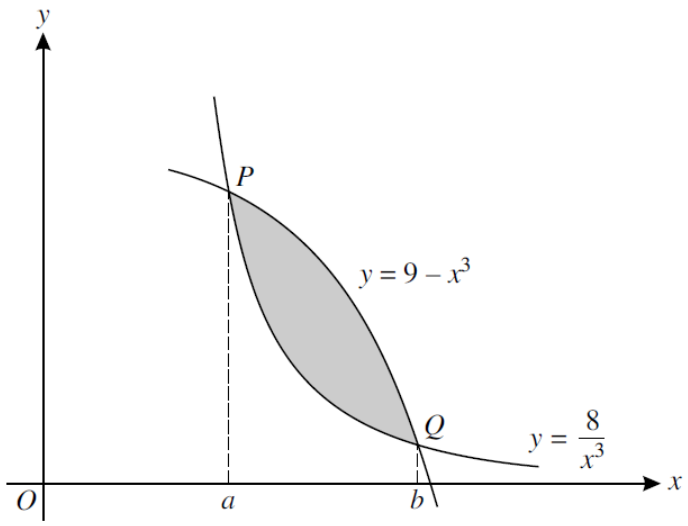
Q10

A curve has equation $y = kx^2 + 1$ and a line has equation $y = kx$, where k is a non-zero constant.

(i) Find the set of values of k for which the curve and the line have no common points. [3]

(ii) State the value of k for which the line is a tangent to the curve and, for this case, find the coordinates of the point where the line touches the curve. [4]

Q11



The diagram shows parts of the curves $y = 9 - x^3$ and $y = \frac{8}{x^3}$ and their points of intersection P and Q. The x-coordinates of P and Q are a and b respectively.

(i) Show that $x = a$ and $x = b$ are roots of the equation $x^6 - 9x^3 + 8 = 0$. Solve this equation and hence state the value of a and the value of b. [4]

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Q12

The equation $x^2 + px + q = 0$, where p and q are constants, has roots -3 and 5.

(i) Find the values of p and q. [2]

(ii) Using these values of p and q, find the value of the constant r for which the equation $x^2 + px + q + r = 0$ has equal roots. [3]

Q13

Find the set of values of m for which the line $y = mx + 4$ intersects the curve $y = 3x^2 - 4x + 7$ at two distinct points. [5]

Q14

A line has equation $y = kx + 6$ and a curve has equation $y = x^2 + 3x + 2k$, where k is a constant.

(i) For the case where $k = 2$, the line and the curve intersect at points A and B. Find the distance AB and the coordinates of the mid-point of AB. [5]

(ii) Find the two values of k for which the line is a tangent to the curve. [4]

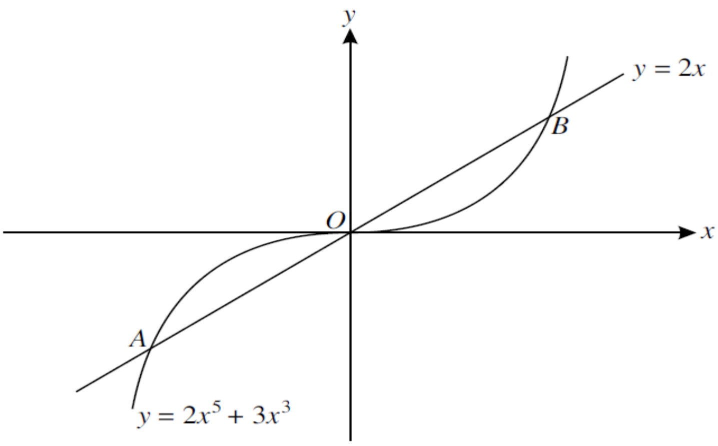
Q15

The equation of a curve is $y^2 + 2x = 13$ and the equation of a line is $2y + x = k$, where k is a constant.

(i) In the case where $k = 8$, find the coordinates of the points of intersection of the line and the curve. [4]

(ii) Find the value of k for which the line is a tangent to the curve. [3]

Q16



The diagram shows the curve $y = 2x^5 + 3x^3$ and the line $y = 2x$ intersecting at points A, O and B.

(i) Show that the x-coordinates of A and B satisfy the equation $2x^4 + 3x^2 - 2 = 0$. [2]

(ii) Solve the equation $2x^4 + 3x^2 - 2 = 0$ and hence find the coordinates of A and B, giving your answers in an exact form. [3]

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Q17

(i) A straight line passes through the point (2, 0) and has gradient m. Write down the equation of the line. [1]

(ii) Find the two values of m for which the line is a tangent to the curve $y = x^2 - 4x + 5$. For each value of m, find the coordinates of the point where the line touches the curve. [6]

(iii) Express $x^2 - 4x + 5$ in the form $(x + a)^2 + b$ and hence, or otherwise, write down the coordinates of the minimum point on the curve. [2]

Q18

The function $f : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geqslant p$, where k and p are constants.

(i) Express $f(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. [2]

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Answers:

Q1:

Eliminates x or y completely
$y^2 - 2y + 2c$ or $4x^2 + x(4c-4) + c^2 = 0$
Use of $b^2 - 4ac = 0$
$\rightarrow \quad c = \frac{1}{2}$
[or gradients equal $2 = 1/\sqrt{x}$ M1A1
$\rightarrow$ value for x, y and c. M1A1]

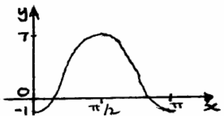
Q2

$\times (x^4) \rightarrow 4x^4 - x^2 - 18 = 0$
$(4x^2 - 9)(x^2 + 2) = 0$
$x = 1.5$ or $x = -1.5$

Q3

$f(x) = a + b\cos 2x$,
$\rightarrow a + b = -1$
and $a - b = 7$
Solution $\rightarrow a = 3$ and $b = -4$

$3 - 4\cos 2x = 0 \rightarrow \cos 2x = \frac{3}{4}$
$\rightarrow x = 0.36$ and 2.78



Q4

1. $y = 4x + k$ and $y = x^2$
$\rightarrow x^2 - 4x - k = 0$
$b^2 - 4ac < 0 \rightarrow 16 + 4k < 0$
$\rightarrow k < -4$
"$2x = 4 \Rightarrow x = 2$" M1 "$y = 4 \Rightarrow k = -4$" M1 $\Rightarrow$ A1

Q5:

$y = x^2 - 3x + 4$

(i) $dy/dx = 2x - 3$
$= 0$ when $x = 1.5$, $y = 1.75$
This is a minimum point, $1.75 > 0$
Curve lies above the x - axis.

(ii) Decreasing function for $x < 1.5$.

(iii) $y = x^2 - 3x + 4$ with $y + 2x = 6$
Eliminate y to give $x^2 - x - 2 = 0$
or eliminate x to give $y^2 - 10y + 16 = 0$

$\rightarrow (-1, 8)$ and $(2, 2)$

(iv) $x^2 - 3x + 4 = k - 2x$
$\rightarrow x^2 - x + 4 - k = 0$ or $2x - 3 = -2$
Use of $b^2 - 4ac = 0$ or $x = \frac{1}{2} \rightarrow y = 2\frac{3}{4}$
$k = 3\frac{3}{4}$

Q6

$kx - 4 = x^2 - 2x \rightarrow x^2 - (2 + k)x + 4 = 0$
Use of $b^2 - 4ac$
$(2 + k)^2 = 16 \quad k = 2$ or -6

$k > 2$ or $k < -6$

Q7

$y = x^2 - 4x + 7 \quad 2y = x + k$
Sim eqns $\rightarrow 2x^2 - 9x + 14 - k = 0$
Uses $b^2 - 4ac$, $81 - 8(14 - k)$
Key value is $k = 3.875$ or $31/8$.
$k < 3.875$

Q8

(i) $2x^2 - 12x + 7 = 2(x-3)^2 - 11$

Q9

$f : x \mapsto 2x^2 - 8x + 14$

(i) $y + kx = 12$, Sim Eqns.
$\rightarrow 2x^2 - 8x + kx + 2 = 0$
Use of $b^2 - 4ac$
$\rightarrow (k - 8)^2 = 16 \rightarrow k = 12$ or 4.

(ii) $2x^2 - 8x + 14 = 2(x - 2)^2 + 6$

Q10

(i) $kx^2 - kx + 1 = 0$
$k^2 - 4k < 0$
$0 < k < 4$

(ii) $k = 4$ only
$(2x - 1)^2 = 0$
$x = \frac{1}{2}$, $y = 2$ or $(\frac{1}{2}, 2)$

Q11

(i) $9 - x^3 = \dfrac{8}{x^3}$

$x^6 - 9x^3 + 8 = 0$
$(X - 1)(X - 8) = 0 \rightarrow X = 1$ or 8
$a = 1, b = 2$

Q12:

(i) $x^2 + px + q = (x + 3)(x - 5)$
$\rightarrow p = -2, \ q = -15.$
(any other method ok)

(ii) $x^2 + px + q + r = 0$
Use of "$b^2 - 4ac$"
Uses a, b and c correctly
$r = 16$

or
$= (x + k)^2 \rightarrow 2k = p$ (M1) $k^2 = q + r$ (M1)
$\rightarrow k = -1 \rightarrow r = 16$ (A1)

Q13

$y = mx + 4 \qquad y = 3x^2 - 4x + 7$
Equate $\rightarrow 3x^2 - (4 + m)x + 3 = 0$
Uses $b^2 - 4ac \rightarrow (4 + m)^2 - 36$
Solution of quadratic $m = 2$ or -10
Set of values $m > 2$ or $m < -10$

Q14

(i) $x^2 + 3x + 4 = 2x + 6 \Rightarrow x^2 + x - 2 (= 0)$
$(x - 1)(x + 2) = 0 \rightarrow (1, 8), (-2, 2)$
$AB = \sqrt{3^2 + 6^2} = 6.71$ or $\sqrt{45}$ or $3\sqrt{5}$
$\left(-\frac{1}{2}, 5\right)$

(ii) $x^2 + (3 - k)x + 2k - 6 (= 0)$
$(3 - k)^2 - 4(2k - 6) = 0$
$(3 - k)(11 - k) = 0$
$k = 3$ or 11

Q15:

(i) $y^2 + 2x = 13, \quad 2y + x = 8$
$\rightarrow y^2 - 4y + 3 = 0, \ x^2 - 8x + 12 = 0$
$\rightarrow (2, 3)$ and $(6, 1)$

(ii) Removes $x \rightarrow y^2 + 2(k - 2y) = 13$
Uses $b^2 - 4ac$ on "quadratic = 0)
$\rightarrow k = 8\frac{1}{2}$
or $\dfrac{dy}{dx} = -\frac{1}{2} = \dfrac{-1}{y} \rightarrow y = 2, x = 4\frac{1}{2}, k = 8\frac{1}{2}$

Q16

(i) $2x^5 + 3x^2 = 2x \Rightarrow 2x^5 + 3x^2 - 2x = 0$
$[x(2x]^4 + 3x^2 - 2) = 0$
$2x^4 + 3x^2 - 2 = 0$

(ii) $(x^2 + 2)(2x^2 - 1) = 0$

$x = \pm \frac{1}{\sqrt{2}}$ only

$\left(\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right), \left(\frac{-1}{\sqrt{2}}, \frac{-2}{\sqrt{2}}\right)$

Q17

(i) $y = m(x - 2)$ oe

(ii) $x^2 - 4x + 5 = mx - 2m \Rightarrow x^2 - x(4 + m) + 5 + 2m = 0$
$(4 + m)^2 - 4(5 + 2m) = 0 \Rightarrow m^2 - 4 = 0$
$m = \pm 2$
$m = 2 \Rightarrow x^2 - 6x + 9 = 0 \Rightarrow x = 3$

$m = -2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1$
$(3, 2), \quad (1, 2)$

OR $m = 2^x - 4$

$y = m^x - 2m$, $y = x2 - 4x + 5$

(iii) $(x - 2)^2 + 1, (2, 1)$

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Q18:

(i) $(x-2)^2 - 4 + k$