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Integration P1

Q1

Find $\int \left(4x + \frac{6}{x^2}\right) \mathrm{d}x$. [3]

Q2

The equation of a curve is $y = \sqrt{(5x + 4)}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

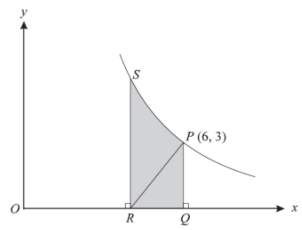
(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of x has the constant value 0.03 units per second. Find the rate of increase of y at the instant when $x = 1$. [2]

(iii) Find the area enclosed by the curve, the x-axis, the y-axis and the line $x = 1$. [5]

Q3

Evaluate $\int_0^1 \sqrt{(3x + 1)}\, \mathrm{d}x$. [4]

Q4

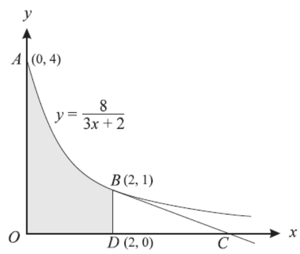


The diagram shows part of the graph of $y = \frac{18}{x}$ and the normal to the curve at $P(6, 3)$. This normal meets the x-axis at R. The point Q on the x-axis and the point S on the curve are such that PQ and SR are parallel to the y-axis.

(i) Find the equation of the normal at P and show that R is the point $(4\frac{1}{2}, 0)$. [5]

(ii) Show that the volume of the solid obtained when the shaded region $PQRS$ is rotated through 360° about the x-axis is 18π. [4]

Q5



The diagram shows points $A(0, 4)$ and $B(2, 1)$ on the curve $y = \frac{8}{3x + 2}$. The tangent to the curve at B crosses the x-axis at C. The point D has coordinates $(2, 0)$.

(i) Find the equation of the tangent to the curve at B and hence show that the area of triangle BDC is $\frac{4}{3}$. [6]

(ii) Show that the volume of the solid formed when the shaded region $ODBA$ is rotated completely about the x-axis is 8π. [5]

Q6

A curve has equation $y = x^2 + \frac{2}{x}$.

(i) Write down expressions for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(iii) Find the volume of the solid formed when the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated completely about the x-axis. [6]

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Q7

A curve has equation $y = \dfrac{4}{\sqrt{x}}$.

(i) The normal to the curve at the point (4, 2) meets the x-axis at P and the y-axis at Q. Find the length of PQ, correct to 3 significant figures. [6]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$. [4]

Q8

A curve is such that $\dfrac{dy}{dx} = \dfrac{16}{x^3}$, and (1, 4) is a point on the curve.

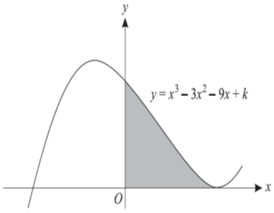
(i) Find the equation of the curve. [4]

(ii) A line with gradient $-\frac{1}{2}$ is a normal to the curve. Find the equation of this normal, giving your answer in the form $ax + by = c$. [4]

(iii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$. [4]

Q9

The diagram shows the curve $y = x^3 - 3x^2 - 9x + k$, where k is a constant. The curve has a minimum point on the x-axis.



(i) Find the value of k. [4]

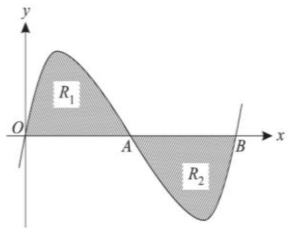
(ii) Find the coordinates of the maximum point of the curve. [1]

(iii) State the set of values of x for which $x^3 - 3x^2 - 9x + k$ is a decreasing function of x. [1]

(iv) Find the area of the shaded region. [4]

Q10

The diagram shows the curve $y = x(x-1)(x-2)$, which crosses the x-axis at the points O (0, 0), A (1, 0) and B (2, 0).



(i) The tangents to the curve at the points A and B meet at the point C. Find the x-coordinate of C. [5]

(ii) Show by integration that the area of the shaded region R_1 is the same as the area of the shaded region R_2. [4]

Q11

The equation of a curve is $y = \dfrac{6}{5 - 2x}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of y has a constant value of 0.02 units per second. Find the rate of increase of x when $x = 1$. [2]

(iii) The region between the curve, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through 360° about the x-axis. Show that the volume obtained is $\frac{12}{5}\pi$. [5]

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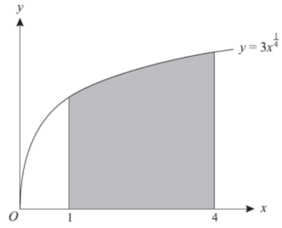
Q12

The diagram shows the curve $y = 3x^{\frac{1}{4}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$. Find the volume of the solid obtained when this shaded region is rotated completely about the x-axis, giving your answer in terms of π. [4]



Q13

The equation of a curve is $y = 2x + \frac{8}{x^2}$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve and determine the nature of the stationary point. [3]

(iii) Show that the normal to the curve at the point $(-2, -2)$ intersects the x-axis at the point $(-10, 0)$. [3]

(iv) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$. [3]

Q14

Find the area of the region enclosed by the curve $y = 2\sqrt{x}$, the x-axis and the lines $x = 1$ and $x = 4$. [4]

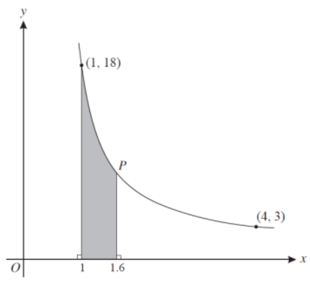
Q15

The diagram shows a curve for which $\frac{dy}{dx} = -\frac{k}{x^3}$, where k is a constant. The curve passes through the points $(1, 18)$ and $(4, 3)$.

(i) Show, by integration, that the equation of the curve is $y = \frac{16}{x^2} + 2$. [4]

The point P lies on the curve and has x-coordinate 1.6.

(ii) Find the area of the shaded region. [4]



Q16

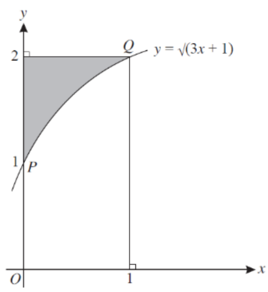
The diagram shows the curve $y = \sqrt{(3x + 1)}$ and the points $P\,(0, 1)$ and $Q\,(1, 2)$ on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 2$.

(i) Find the area of the shaded region. [4]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

Tangents are drawn to the curve at the points P and Q.

(iii) Find the acute angle, in degrees correct to 1 decimal place, between the two tangents. [4]



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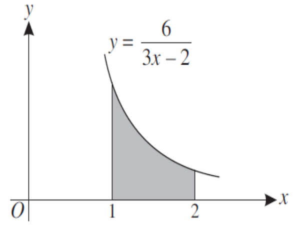
Q17

The diagram shows part of the curve $y = \frac{6}{3x-2}$.

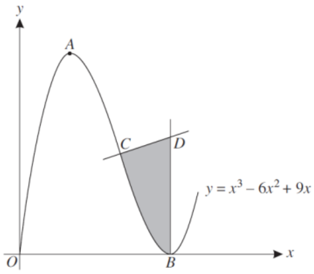


(i) Find the gradient of the curve at the point where $x = 2$. [3]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis, giving your answer in terms of π. [5]

Q18

The diagram shows the curve $y = x^3 - 6x^2 + 9x$ for $x \geqslant 0$. The curve has a maximum point at A and a minimum point on the x-axis at B. The normal to the curve at C (2, 2) meets the normal to the curve at B at the point D.



(i) Find the coordinates of A and B. [3]

(ii) Find the equation of the normal to the curve at C. [3]

(iii) Find the area of the shaded region. [5]

Q19

The equation of a curve is $y = x^4 + 4x + 9$.

(i) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 1$. [3]

Q20

The function f is such that $f(x) = \frac{3}{2x+5}$ for $x \in \mathbb{R}, x \neq -2.5$.

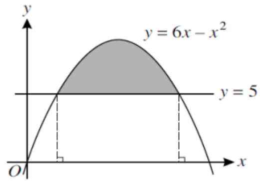
(i) Obtain an expression for $f'(x)$ and explain why f is a decreasing function. [3]

(ii) Obtain an expression for $f^{-1}(x)$. [2]

(iii) A curve has the equation $y = f(x)$. Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 2$ is rotated through 360° about the x-axis. [4]

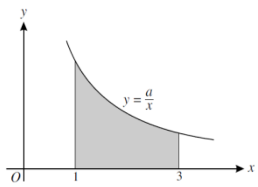
Q21

The diagram shows the curve $y = 6x - x^2$ and the line $y = 5$. Find the area of the shaded region. [6]



Q22

The diagram shows part of the curve $y = \frac{a}{x}$, where a is a positive constant. Given that the volume obtained when the shaded region is rotated through 360° about the x-axis is 24π, find the value of a. [4]



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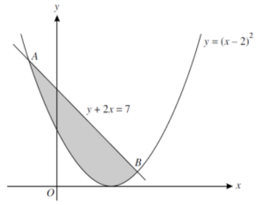
Q23

The diagram shows the curve $y = (x - 2)^2$ and the line $y + 2x = 7$, which intersect at points A and B. Find the area of the shaded region. [8]

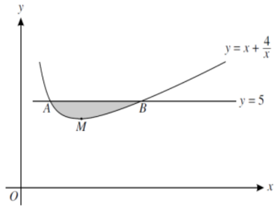


Q24

The diagram shows part of the curve $y = x + \frac{4}{x}$ which has a minimum point at M. The line $y = 5$ intersects the curve at the points A and B.

(i) Find the coordinates of A, B and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through $360°$ about the x-axis. [6]



Q25

Find $\int \left(x + \frac{1}{x}\right)^2 \mathrm{d}x$. [3]

Q26

The equation of a curve is $y = \frac{9}{2 - x}$.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and determine, with a reason, whether the curve has any stationary points. [3]

(ii) Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 1$ is rotated through $360°$ about the x-axis. [4]

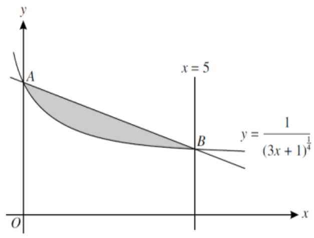
(iii) Find the set of values of k for which the line $y = x + k$ intersects the curve at two distinct points. [4]

Q27

The diagram shows part of the curve $y = \frac{1}{(3x + 1)^{\frac{1}{4}}}$. The curve cuts the y-axis at A and the line $x = 5$ at B.

(i) Show that the equation of the line AB is $y = -\frac{1}{10}x + 1$. [4]

(ii) Find the volume obtained when the shaded region is rotated through $360°$ about the x-axis. [9]



Q28

A curve has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = 3x^2 + 2x - 5$.

(i) Find the set of values of x for which f is an increasing function. [3]

(ii) Given that the curve passes through $(1, 3)$, find $\mathrm{f}(x)$. [4]

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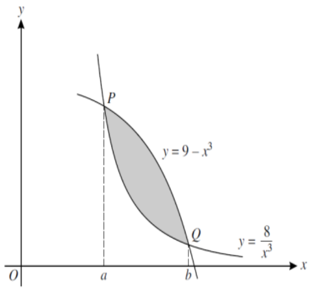
Q29

The diagram shows parts of the curves $y = 9 - x^3$ and $y = \frac{8}{x^3}$ and their points of intersection P and Q. The x-coordinates of P and Q are a and b respectively.



(i) Show that $x = a$ and $x = b$ are roots of the equation $x^6 - 9x^3 + 8 = 0$. Solve this equation and hence state the value of a and the value of b. [4]

(ii) Find the area of the shaded region between the two curves. [5]

(iii) The tangents to the two curves at $x = c$ (where $a < c < b$) are parallel to each other. Find the value of c. [4]

Q30

(i) Sketch the curve $y = (x - 2)^2$. [1]

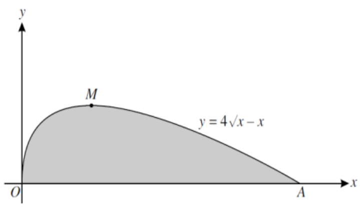
(ii) The region enclosed by the curve, the x-axis and the y-axis is rotated through 360° about the x-axis. Find the volume obtained, giving your answer in terms of π. [4]

Q31

Find $\int \left(x^3 + \frac{1}{x^3}\right) dx$. [3]

Q32

The diagram shows part of the curve $y = 4\sqrt{x} - x$. The curve has a maximum point at M and meets the x-axis at O and A.



(i) Find the coordinates of A and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis, giving your answer in terms of π. [6]

Q33

Find $\int (3x - 2)^5 \, dx$ and hence find the value of $\int_0^1 (3x - 2)^5 \, dx$. [4]

Q34

A function f is defined for $x \in \mathbb{R}$ and is such that $f'(x) = 2x - 6$. The range of the function is given by $f(x) \geqslant -4$.

(i) State the value of x for which $f(x)$ has a stationary value. [1]

(ii) Find an expression for $f(x)$ in terms of x. [4]

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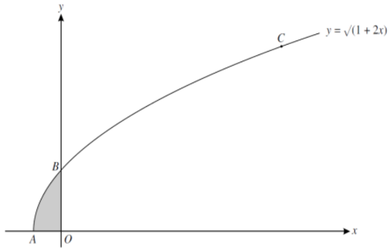
Q35



The diagram shows the curve $y = \sqrt{(1 + 2x)}$ meeting the x-axis at A and the y-axis at B. The y-coordinate of the point C on the curve is 3.

(i) Find the coordinates of B and C. [2]

(ii) Find the equation of the normal to the curve at C. [4]

(iii) Find the volume obtained when the shaded region is rotated through 360° about the **y-axis**. [5]

Q36

A curve is such that $\frac{dy}{dx} = 5 - \frac{8}{x^2}$. The line $3y + x = 17$ is the normal to the curve at the point P on the curve. Given that the x-coordinate of P is positive, find

(i) the coordinates of P, [4]

(ii) the equation of the curve. [4]

Q37

The equation of a curve is $y = \sqrt{(8x - x^2)}$. Find

(i) an expression for $\frac{dy}{dx}$, and the coordinates of the stationary point on the curve, [4]

(ii) the volume obtained when the region bounded by the curve and the x-axis is rotated through 360° about the x-axis. [4]

Q38

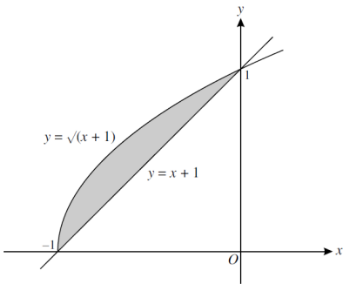
A curve $y = f(x)$ has a stationary point at $P\,(3, -10)$. It is given that $f'(x) = 2x^2 + kx - 12$, where k is a constant.

(i) Show that $k = -2$ and hence find the x-coordinate of the other stationary point, Q. [4]

(ii) Find $f''(x)$ and determine the nature of each of the stationary points P and Q. [2]

(iii) Find $f(x)$. [4]

Q39



The diagram shows the line $y = x + 1$ and the curve $y = \sqrt{(x + 1)}$, meeting at $(-1, 0)$ and $(0, 1)$.

(i) Find the area of the shaded region. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the **y-axis**. [7]

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Answers:

Q1: $\int = 2x^2 - 6x^{-1} + c$

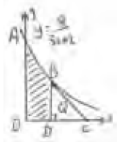
Q2: (i) 5/6 (ii) 0.025 (iii) 2.53

Q3: 1.56

Q4: (i) $y = 2x - 9$ (ii) 18π

Q5:

9



(i) $dy/dx = -24/(3x+2)^2$

Eqn of tangent $y-1 = -\frac{3}{8}(x-2)$
Cuts $y=0$ when $x = 4\frac{2}{3}$

Area of Q $= \frac{1}{2} \times 2\frac{2}{3} \times 1 = \frac{4}{3}$

(ii) Vol $= \pi \int y^2 dx = \pi \int 64(3x+2)^{-2} dx$
$= \pi [-64(3x+2)^{-1} \div 3]$
Limits from 0 to 2
$\rightarrow 8\pi$

Q6:

(i) $dy/dx = 2x - 2/x^2$

$d^2y/dx^2 = 2 + 4/x^3$

(ii) $dy/dx = 0 \quad 2x - 2/x^2 = 0$
$\rightarrow x^3 = 1 \rightarrow x = 1, y = 3$

If $x = 1$, $d^2y/dx^2 > 0$, Minimum

(iii) Vol $= \pi \int y^2 dx = \pi \int (x^4 + 4/x^2 + 4x)\, dx$

$= \pi [x^5/5 - 4/x + 2x^2]$

$[\ ]_2 - [\ ]_1 = \quad 71\pi/5$ or 44.6

Q7:

$y = \frac{4}{\sqrt{x}}$

(i) $dy/dx = -2x^{-1.5}$

$= -\frac{1}{4}$
m of normal $= 4$
Eqn of normal $y - 2 = 4(x - 4)$
P (3.5, 0) and Q (0, −14)

Length of $PQ = \sqrt{3.5^2 + 14^2}$
$= 14.4$

(ii) Area $= \int_1^4 4x^{-0.5} dx = [\frac{4x^{0.5}}{0.5}]$

$= [8\sqrt{x}\,] = 16 - 8 = 8$

Q8: $\frac{dy}{dx} = \frac{16}{x^3}$, through (1,4)

(i) $y = \frac{16x^{-2}}{-2} + c$

Use of (1,4) $\rightarrow$ $y = \frac{-8}{x^2} + 12$

(ii) Normal has $m = -\frac{1}{2}$,
Perpendicular = 2
$\frac{16}{x^3} = 2 \quad x = 2$
$y = 10$
Eqn of normal $y - 10 = -\frac{1}{2}(x - 2)$
ie $2y + x = 22$

(iii) A $= \int (\frac{-8}{x^2} + 12)dx$

$= \frac{8}{x} + 12x$
$= [\ ]$ at 2 − $[\ ]$ at 1

$\rightarrow 8$

Q9:

0. $y = x^3 - 3x^2 - 9x + k$

(i) $dy/dx = 3x^2 - 6x - 9$
$= 0$ when $x = 3$ or $x = -1$
$\rightarrow x = 3, y = 0 \rightarrow k = 27$

(ii) $x = -1 \rightarrow y = 32$

(iii) $-1 < x < 3$.

(iv) Integrate y to get area.
$\rightarrow [\frac{x^4}{4} - x^3 - \frac{9x^2}{2} + kx\ ]$
$\rightarrow 33.75$ when $x = 3$.

Q10:

(i) $y = x^3 - 3x^2 + 2x$
$\frac{dy}{dx} = 3x^2 - 6x + 2$
At A (1,0), $m = -1 \rightarrow y = -1(x-1)$
At B (2,0), $m = 2 \rightarrow y = 2(x-2)$
Sim equations $\rightarrow x = \frac{5}{3}$

(ii) $R_1 = \int_0^1 (x^3 - 3x^2 + 2x)dx$

$= \left[\frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2}\right] = \frac{1}{4}$

$R_2 = [\ ]^2 - [\ ]^1 = -\frac{1}{4}$

Q11:

$y = \frac{6}{5 - 2x}$

(i) $\frac{dy}{dx} = -6(5-2x)^{-2} \times (-2)$

$= \frac{12}{(5-2x)^2} \rightarrow \frac{4}{3}$

(ii) Use of chain rule.
$\frac{dx}{dt} = 0.02 \div (i) = 0.015$

(iii) $V = \pi \int \left(\frac{36}{(5-2x)^2}\right) dx$

$= 36\pi \left[\frac{(5-2x)^{-1}}{-1}\right] \div (-2)$

$[\ ]^1 - [\ ]^0 = \frac{12\pi}{5}$

Q12:

$V = \pi \int 9\sqrt{x} dx$

$= \pi \frac{9x^{\frac{3}{2}}}{\frac{3}{2}}$

$[\ ]$ at 4 − $[\ ]$ at 1 $\rightarrow 42\pi$

Q13:

(i) $\frac{dy}{dx} = 2 - \frac{16}{x^3}$
$\frac{d^2y}{dx^2} = \frac{48}{x^4}$

(ii) $\frac{dy}{dx} = 0 \quad \rightarrow \quad x = 2, y = 6.$
$\frac{d^2y}{dx^2}$ is +ve Minimum.

(iii) $x = -2 \quad m = 4$
Perp gradient $= -\frac{1}{4}$
$y + 2 = -\frac{1}{4}(x + 2)$
Sets y to $0 \rightarrow x = -10$

(iv) Area $= \left[x^2 - \frac{8}{x}\right]$
Evaluated from 1 to 2 $\rightarrow 7$

Q14:

2. Area = integral of $2\sqrt{x}$ attempted.
$\rightarrow \frac{2x^{1.5}}{1.5}$
Uses limits 1 to 4 correctly
$\rightarrow \frac{32}{3} - \frac{4}{3} = 9\frac{1}{3}$ or 9.33 or $\frac{28}{3}$

Q15:

$\frac{dy}{dx} = \frac{k}{x^3}$

(i) Integrating $y = -k\frac{x^{-2}}{-2}$ (+ c)

Sub (1,18) $18 = \frac{k}{2} + c$

Sub (4,3) $3 = \frac{k}{32} + c$

$\rightarrow k = 32, c = 2$

(ii) Area $= \left[-\frac{16}{x} + 2x\right]$ from 1 to 1.6

$\rightarrow [-10 + 3.2] - [-16 + 2] = 7.2$

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Q16:

9 $y = \sqrt{3x+1}$

(i) $A = \int x \, dy = \int_1^2 \frac{y^2-1}{3} dy$

$= \left[\frac{y^3}{9} - \frac{y}{3}\right] = \frac{4}{9}$ (allow 0.44 to 0.45)

[or $2 - \int \sqrt{3x+1}\, dx = [2 - \frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2}\times 3}] = \frac{4}{9}$]

B1 B1 M1A1

(ii) $V = \pi \int y^2 dx = \pi \int (3x+1) dx$

$= \pi\left(\frac{3x^2}{2} + x\right)$ from 0 to 1

Vol of cylinder $= \pi \times 2^2 \times 1 = 4\pi$
→ Subtraction → 1.5π (4.71)

(iii) $\frac{dy}{dx} = \frac{1}{2}(3x+1)^{-\frac{1}{2}} \times 3$

If $x = 0$, $m = \frac{3}{2}$. If $x = 1$, $m = \frac{3}{4}$

At $x = 0$, angle = 56.3°
At $x = 1$, angle = 36.9°
→ angle between = 19.4°

Could use vectors, or $\tan(A - B)$ formula.
Could also find tangents, point of intersection, 3 lengths and cosine rule.

Q17:

(i) $dy/dx = -6(3x-2)^{-2} \times 3$
If $x = 2$, $m = -1\frac{1}{8}$ (−1.125)

(ii) Vol $= \int \frac{36}{(3x-2)^2} dx$

$[\frac{-36}{(3x-2)} \div 3]$

Use of limits [2] − [1] → 9π

Q18:

(i) $\frac{dy}{dx} = 3x^2 - 12x + 9$

Solves $\frac{dy}{dx} = 0$

→ $A(1, 4)$, $B(3, 0)$.

(ii) If $x = 2$, $m = -3$
Normal has $m = \frac{1}{3}$
Eqn $y - 2 = \frac{1}{3}(x-2)$ or $3y = x + 4$.

(iii) area under curve – integrate y.
→ $\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2$
Limits 2 to "his 3" → ¾ (0.75)

Area of trapezium = ½ × 1 × (2 + 2⅓)
$= 2\frac{1}{6}$

Subtract → shaded area of $1\frac{5}{12}$

Q19:

$y = x^4 + 4x + 9$

(i) Differential $= 4x^3 + 4$
Sets to 0 + solution → (−1, 6)
2nd differential $= 12x^2$
Positive, → Minimum

(ii) $A = \left[\frac{x^5}{5} + 2x^2 + 9x\right]$
Limits from 0 to 1 → 11.2

Q20:

$x \mapsto \frac{3}{2x+5}$

(i) $f'(x) = -3(2x+5)^{-2} \times 2$
$f'(x)$ is negative → decreasing

(ii) $y = \frac{3}{2x+5} \to 2x + 5 = \frac{3}{y}$
$\to f^{-1}(x) = \frac{1}{2}\left(\frac{3}{x} - 5\right)$ or $\frac{3-5x}{2x}$

(iii) $\int \pi \frac{9}{(2x+5)^2} dx$
$= (-9\pi(2x+5)^{-1} \div 2)$
Limits 0 to 2 → $\pi(-½ - -0.9)$
→ $= 0.4\pi$ (or 1.26)

Q21:

$y = 6x - x^2$
Meets $y = 5$ when $x = 1$ or $x = 5$.
Integral $= 3x^2 - \frac{1}{3}x^3$
Their limits (1 to 5) used → 30⅔
Area of rectangle = 20
Shaded area = 10⅔

(integral of $6x - x^2 - 5$ B1, M1, A1
DM1 as above, then "$-5x$" B1√ A1)

Q22:

$y = \frac{a}{x}$

Volume $= \pi \int \left(\frac{a^2}{x^2}\right) dx = (\pi)\left[\frac{-a^2}{x}\right]$

Use of limits 1 to 3 → $\frac{2\pi a^2}{3}$

Equates to 24π → $a = 6$

Q23:

$y = (x-2)^2$ and $y + 2x = 7$
Elimination of y → $x^2 - 2x - 3 = 0$
→ $A(-1, 9)$ and $B(3, 1)$

Area under line = ½ × 4 × 10
or $\left[7x - x^2\right]$ from −1 to 3.

Area under curve $= \left[\frac{(x-2)^3}{3}\right]$

or $\left[\frac{x^3}{3} - 2x^2 + 4x\right]$ from −1 to 3

→ 10⅔.
[ok to use $\int (y_1 - y_2) dx$ – marks the same]

Q24:

$y = x + \frac{4}{x}$

(i) $x + \frac{4}{x} = 5 \to A(1, 5), B(4, 5)$

$\frac{dy}{dx} = 1 - \frac{4}{x^2}$

$= 0$ when $x = 2$, $M(2, 4)$.

(ii) Vol of cylinder $= \pi 5^2.3$
Vol under curve $= \pi \int y^2 dx$

Integral $= \frac{x^3}{3} - \frac{16}{x} + 8x$

Uses his limits "1 to 4"
→ $75\pi - 57\pi = 18\pi$

Q25:

$\int \left(x + \frac{1}{x}\right)^2 dx$

$= \frac{x^3}{3} - \frac{1}{x} + 2x + (c)$

Q26:

$y = \frac{9}{2-x}$

(i) $\frac{dy}{dx} = -9(2-x)^{-2} \times -1$

$\frac{9}{(2-x)^2} \neq 0$. No turning points.

(ii) $V = \pi \int \frac{81}{(2-x)^2} dx$

$\int y^2 dx = -81(2-x)^{-1} \div (-1)$

Use of limits 0 to 1

→ $\frac{81\pi}{2}$ (or 127)

(iii) $\frac{9}{2-x} = x + k$
→ $x^2 - 2x + kx - 2k + 9 = 0$
Uses $b - 4ac$
→ $k^2 + 4k - 32$
→ end-points of 4 and −8
Range for 2 points of intersection
→ $k < -8$, $k > 4$.

Q27:

(i) $A = (0, 1)$
$B = (5, ½)$

$y - 1 = -\frac{1}{10}(x - 0)$

$y = -\frac{1}{10}x + 1$

(ii) Curve: $(\pi)\int_0^5 (3x+1)^{-1/2} dx$

$\frac{2\pi}{3}[(3x+1)^{½}]_0^5$

$\frac{2\pi}{3}[4-1]$

$[2\pi]$

Line: $(\pi)\int_0^5 (\frac{1}{100}x^2 - \frac{1}{5}x + 1)dx$

$(\pi)[\frac{1}{300}x^3 - \frac{1}{10}x^2 + x]_0^5$

$(\pi)[\frac{125}{300} - \frac{25}{10} + 5]$

$[\frac{35\pi}{12}]$

Volume $= \frac{35\pi}{12} - 2\pi = \frac{11\pi}{12}$

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Q28:

(i) $(3x+5)(x-1)(>0)$
$-5/3,\ 1$
$x < -5/3,\ x > 1$

(ii) $f(x) = x^3 + x^2 - 5x\ (+c)$

$3 = 1 + 1 - 5 + c$
$f(x) = x^3 + x^2 - 5x + 6$

Q29:

(i) $9 - x^3 = \frac{8}{x^3}$
$x^6 - 9x^3 + 8 = 0$
$(X-1)(X-8) = 0 \rightarrow X = 1$ or 8
$a = 1,\ b = 2$

(ii) $\int_1^2 \left[(9 - x^3) - \frac{8}{x^3}\right]dx$

$\left[9x - \frac{x^4}{4}\right] \cdot \left[\frac{-4}{x^2}\right]$

$18 - 4 + 1 - (9 - \frac{1}{4} + 4)$

$2\frac{1}{4}$

(iii) $\frac{dy}{dx} = \frac{-24}{x^4},\ \frac{dy}{dx} = -3x^2$
$\frac{-24}{c^4} = -3c^2$
$c^6 = 8$
$c = \sqrt{2}$ or $8^{1/6}$ or $1.41(4\ldots)$

Q30:

(i) Correct shape – touching positive x-axis

(ii) $(\pi)\int(x-2)^4 dx$

$(\pi)\left[\frac{(x-2)^5}{5}\right]$

$(\pi)[0 - (-32)/5]$

$\frac{32\pi}{5}$ or 6.4π

Q31:

$$\int\left(x^3 + \frac{1}{x^3}\right)dx = \frac{x^4}{4} + \frac{x^{-2}}{-2} + c$$

Q32:

$y = 4\sqrt{x} - x.$

(i) At A, $4\sqrt{x} - x = 0 \rightarrow A(16, 0)$
$\frac{dy}{dx} = 2x^{-\frac{1}{2}} - 1$
$= 0$ when $x = 4 \rightarrow (4, 4)$

(ii) Vol $= \pi\int y^2\ dx =$

$\pi\int(16x + x^2 - 8x^{\frac{3}{2}})\ dx$

$\pi[8x^2 + \frac{x^3}{3} - 8\frac{x^{\frac{5}{2}}}{\frac{5}{2}}]$

Limits 0 to 16 $\rightarrow$ 136.5π. (or 137π)

Q33:

$\int(3x-2)^5 dx = \frac{(3x-2)^6}{6} \div 3\ (+c)$

$\int_0^1 (3x-2)^5 dx = \left[\frac{(3x-2)^6}{18}\right]$

Limits used correctly $\rightarrow -3\frac{1}{2}$

Q34:

(i) 3

(ii) $f(x) = x^2 - 6x(+c)$

Subst $(3, -4)$

$c = 5 \rightarrow f(x) = x^2 - 6x + 5$

Q35:

(i) $B = (0,1)$ $C = (4,3)$

(ii) $\frac{\delta y}{\delta x} = \frac{1}{2} \times 2(1+2x)^{-\frac{1}{2}}$

Grad. of normal $= -3$
$y - 3 = -3(x-4)$ or $y = -3x + 15$ oe

(iii) $y^2 = 1 + 2x \Rightarrow x = \frac{1}{2(y^2 - 1)}$ SOI

$(\pi) \times \frac{1}{4} \times \int(y^4 - 2y^2 + 1)\delta y$

$(\pi) \times \frac{1}{4}\left[\frac{y^5}{5} - \frac{2y^3}{3} + y\right]$

$(\pi) \times \frac{1}{4}\left[\frac{1}{5} - \frac{2}{3} + 1\right]$

$\frac{2}{15}\pi$

Q36:

$\frac{dy}{dx} = 5 - \frac{8}{x^2}$, Normal $3y + x = 17$

(i) Gradient of line $= -\frac{1}{3}$
$\frac{dy}{dx} = 3 \rightarrow x = 2,\ y = 5$

(ii) $y = 5x + 8x^{-1}(+c)$
Uses $(2, 5) \rightarrow c = -9$

Q37:

$y = \sqrt{8x - x^2}$

(i) $\frac{dy}{dx} = \frac{1}{2}(8x - x^2)^{-\frac{1}{2}} \times (8 - 2x)$
$= 0$ when $x = 4.$
$\rightarrow (4, 4)$

(ii) $y = 0$ when $x = 0$ or 8
Vol $= \pi\int(8x - x^2)dx$

$= \pi\left[4x^2 - \frac{x^3}{3}\right]$

$\rightarrow \frac{256\pi}{3}$

Q38:

(i) $f'(3) = 0 \Rightarrow 18 + 3k - 12 = 0$
$k = -2$
$(x-3)(x+2) = 0$
$x = -2$, (Allow also $= 3$)

(ii) $f''(x) = 4x - 2$
$f''(3) > 0$ hence min at P
$f''(-2) < 0$ hence max at Q

(iii) $f(x) = \frac{2}{3}x^3 - x^2 - 12x\ (+c)$
Sub $(3, -10) \rightarrow -10 = 18 - 9 - 36 + c$
$c = 17$

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Q39:

(i) $\int (x+1)^{\frac{1}{2}} - (x+1)$ or $\int (y^2-1)-(y-1)$

$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{1}{2}x^2 - x$ or $\frac{1}{3}y^3 - \frac{1}{2}y^2$

$\frac{2}{3} - \left(0 - \frac{1}{2} + 1\right)$ or $\frac{1}{3} - \frac{1}{2}$

$\frac{1}{6}$

(ii) $V_1 = (\pi)\int (y^2-1)^2 = (\pi)\int y^4 - 2y^2 + 1$

$(\pi)\left[\frac{y^5}{5} - \frac{2y^3}{3} + y\right]$

$(\pi)\left[\frac{1}{5} - \frac{2}{3} + 1\right]$

$V_1 = \frac{8}{15(\pi)}$ or $0.533(\pi)$ (AWRT)

or $(\pi)\left[y\uparrow 3/3 - y\uparrow 2 + y\right]$

$V_2 = \frac{1}{3}\pi$

Volume $= \frac{8}{15}\pi \ \frac{1}{-3}\pi = \frac{1}{5}\pi$ (or 0.628)

OR $(y^4 - 2y^2 + 1) - (y^2 - 2y + 1)$

$(\pi)\int y^4 - 3y^2 + 2y$

$(\pi)\left[y\uparrow 5/5 - y\uparrow 3 + y\uparrow 2\right]$

$(\pi)\left[\frac{1}{5} - 1 + 1\right]$

$\frac{1}{5}\pi$