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Algebra – Factor Remainder Theorem P3

Q1

The polynomial $x^3 - 2x + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

(ii) When a has this value, find the quadratic factor of $\mathrm{p}(x)$. [2]

Q2

The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $x^2 + x + 2$ is a factor of $\mathrm{p}(x)$. Find the value of a and the other quadratic factor of $\mathrm{p}(x)$. [4]

Q3

The polynomial $4x^3 - 4x^2 + 3x + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $\mathrm{p}(x)$ is divisible by $2x^2 - 3x + 3$.

(i) Find the value of a. [3]

(ii) When a has this value, solve the inequality $\mathrm{p}(x) < 0$, justifying your answer. [3]

Q4

The polynomial $2x^3 + ax^2 + bx - 4$, where a and b are constants, is denoted by $\mathrm{p}(x)$. The result of differentiating $\mathrm{p}(x)$ with respect to x is denoted by $\mathrm{p}'(x)$. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$ and of $\mathrm{p}'(x)$.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

Q5

The polynomial $2x^3 + 5x^2 + ax + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is 9.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

Q6

The polynomial $\mathrm{p}(z)$ is defined by

$$\mathrm{p}(z) = z^3 + mz^2 + 24z + 32,$$

where m is a constant. It is given that $(z + 2)$ is a factor of $\mathrm{p}(z)$.

(i) Find the value of m. [2]

(ii) Hence, showing all your working, find

(a) the three roots of the equation $\mathrm{p}(z) = 0$, [5]

(b) the six roots of the equation $\mathrm{p}(z^2) = 0$. [6]

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Q7

The polynomial $f(x)$ is defined by

$$f(x) = 12x^3 + 25x^2 - 4x - 12.$$

(i) Show that $f(-2) = 0$ and factorise $f(x)$ completely. [4]

(ii) Given that

$$12 \times 27^y + 25 \times 9^y - 4 \times 3^y - 12 = 0,$$

state the value of 3^y and hence find y correct to 3 significant figures. [3]

Q8

The polynomial $ax^3 + bx^2 + 5x - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x - 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the quadratic factor of $p(x)$. [2]

Q9

The polynomial $x^4 + 3x^3 + ax + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 1$.

(i) Find the value of a. [4]

(ii) When a has this value, find the real roots of the equation $p(x) = 0$. [2]

Q10

The polynomial $p(x)$ is defined by

$$p(x) = ax^3 - x^2 + 4x - a,$$

where a is a constant. It is given that $(2x - 1)$ is a factor of $p(x)$.

(i) Find the value of a and hence factorise $p(x)$. [4]

(ii) When a has the value found in part **(i)**, express $\frac{8x - 13}{p(x)}$ in partial fractions. [5]

Q11

The polynomial $p(x)$ is defined by

$$p(x) = x^3 - 3ax + 4a,$$

where a is a constant.

(i) Given that $(x - 2)$ is a factor of $p(x)$, find the value of a. [2]

(ii) When a has this value,

(a) factorise $p(x)$ completely, [3]

(b) find all the roots of the equation $p(x^2) = 0$. [2]

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Answers

Q1:

(i) Substitute $x = -2$ and equate to zero, or divide by $x + 2$ and equate constant remainder to zero, or use a factor $Ax^2 + Bx + C$ and reach an equation in a
Obtain answer $a = 4$

(ii) Attempt to find quadratic factor by division or inspection
State or exhibit quadratic factor $x^2 - 2x + 2$
[The M1 is earned if division reaches a partial quotient $x^2 + kx$, or if inspection has an unknown factor $x^2 + bx + c$ and an equation in b and/or c, or if inspection without working states two coefficients with the correct moduli.]

Q2:

EITHER: Attempt division by $x^2 + x + 2$ reaching a partial quotient of $x^2 + kx$
Complete the division and obtain quotient $x^2 - x + 2$
Equate constant remainder to zero and solve for a
Obtain answer $a = 4$

Q3:

(i) *EITHER*: Attempt division by $2x^2 - 3x + 3$ and state partial quotient $2x$
Complete division and form an equation for a
Obtain $a = 3$

(ii) State answer $x < -\frac{1}{2}$ only
Carry out a complete method for showing $2x^2 - 3x + 3$ is never zero
Complete the justification of the answer by showing that $2x^2 - 3x + 3 > 0$ for all x
[These last two marks are independent of the B mark, so B0M1A1 is possible. Alternative methods include (a) Complete the square M1 and use a correct completion to justify the answer A1; (b) Draw a recognizable graph of $y = 2x^2 + 3x - 3$ or p(x) M1 and use a correct graph to justify the answer A1; (c) Find the x-coordinate of the stationary point of $y = 2x^2 + 3x - 3$ and either find its y-coordinate or determine its nature M1, then use minimum point with correct coordinates to justify the answer A1.]
[Do not accept $\le$ for $<$]

Q4:

(i) Substitute $x = -2$, equate to zero and state a correct equation, e.g. $-16 + 4a - 2b - 4 = 0$
Differentiate p(x), substitute $x = -2$ and equate to zero
Obtain a correct equation, e.g. $24 - 4a + b = 0$
Solve for a or for b
Obtian $a = 7$ and $b = 4$

(ii) *EITHER*: State or imply $(x + 2)^2$ is a factor
Attempt division by $(x + 2)^2$ reaching a quotient $2x + k$ or use inspection with unknown factor $cx + d$ reaching $c = 2$ or $d = -1$
Obtain factorisation $(x + 2)^2(2x - 1)$

Q5:

(i) Substitute $x = -\frac{1}{2}$, equate to zero and obtain a correct equation, e.g. $-\frac{1}{4} + \frac{5}{4} - \frac{1}{2}a + b = 0$
Substitute $x = -2$ and equate to 9
Obtain a correct equation, e.g. $-16 + 20 - 2a + b = 9$
Solve for a or for b
Obtain $a = -4$ and $b = -3$

(ii) Attempt division by $2x + 1$ reaching a partial quotient of $x^2 + kx$ M1
Obtain quadratic factor $x^2 + 2x - 3$ A1
Obtain factorisation $(2x + 1)(x + 3)(x - 1)$ A1 [3]

[The M1 is earned if inspection has an unknown factor of $x^2 + ex + f$ and an equation in e and/or f, or if two coefficients with the correct moduli are stated without working.]
[If linear factors are found by the factor theorem, give B1 + B1 for $(x - 1)$ and $(x + 3)$, and then B1 for the complete factorisation.]

Q6:

(i) Attempt to solve for m the equation p(−2) = 0 or equivalent
Obtain $m = 6$

Alternative:
Attempt p(z) ÷ (z + 2), equate a constant remainder to zero and solve for m.
Obtain $m = 6$

(ii) (a) State $z = -2$
Attempt to find quadratic factor by inspection, division, identity, …
Obtain $z^2 + 4z + 16$
Use correct method to solve a 3-term quadratic equation
Obtain $-2 \pm 2\sqrt{3}\mathrm{i}$ or equivalent

(b) State or imply that square roots of answers from part **(ii)(a)** needed
Obtain $\pm \mathrm{i}\sqrt{2}$
Attempt to find square root of a further root in the form $x + \mathrm{i}y$ or in polar form
Obtain $a^2 - b^2 = -2$ and $ab = (\pm)\sqrt{3}$ following their answer to part **(ii)(a)**
Solve for a and b
Obtain $\pm\left(1 + \mathrm{i}\sqrt{3}\right)$ and $\pm\left(1 - \mathrm{i}\sqrt{3}\right)$

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Q7:

(i) Verify that $-96 + 100 + 8 - 12 = 0$
Attempt to find quadratic factor by division by $(x + 2)$, reaching a partial quotient $12x^2 + kx$, inspection or use of an identity
Obtain $12x^2 + x - 6$
State $(x + 2)(4x + 3)(3x - 2)$
[The M1 can be earned if inspection has unknown factor $Ax^2 + Bx - 6$ and an equation in A and/or B or equation $12x^2 + Bx + C$ and an equation in B and/or C.]

(ii) State $3^y = \frac{2}{3}$ and no other value
Use correct method for finding y from equation of form $3^y = k$, where $k > 0$
Obtain -0.369 and no other value

Q8:

(i) Substitute $x = \frac{1}{2}$ and equate to zero, or divide, and obtain a correct equation, e.g. $\frac{1}{8}a + \frac{1}{4}b + \frac{5}{2} - 2 = 0$
Substitute $x = 2$ and equate result to 12, or divide and equate constant remainder to 12
Obtain a correct equation, e.g. $8a + 4b + 10 - 2 = 12$
Solve for a or for b
Obtain $a = 2$ and $b = -3$

(ii) Attempt division by $2x - 1$ reaching a partial quotient $\frac{1}{2}ax^2 + kx$
Obtain quadratic factor $x^2 - x + 2$
[The M1 is earned if inspection has an unknown factor $Ax^2 + Bx + 2$ and an equation in A and/or B, or an unknown factor of $\frac{1}{2}ax^2 + Bx + C$ and an equation in B and/or C.]

Q9:

(i) *EITHER*: Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$
Obtain quotient $x^2 + 4x + 3$
Equate remainder of form lx to zero and solve for a, or equivalent
Obtain answer $a = 1$

(ii) State answer, e.g. $x = -3$
State answer, e.g. $x = -1$ and no others

Q10:

(i) Substitute $x = \frac{1}{2}$ and equate to zero
or divide by $(2x - 1)$, reach $\frac{a}{2}x^2 + kx + \ldots$ and equate remainder to zero
or by inspection reach $\frac{a}{2}x^2 + bx + c$ and an equation in b/c
or by inspection reach $Ax^2 + Bx + a$ and an equation in A/B
Obtain $a = 2$
Attempt to find quadratic factor by division or inspection or equivalent
Obtain $(2x - 1)(x^2 + 2)$

(ii) State or imply form $\frac{A}{2x-1} + \frac{Bx+C}{x^2+2}$, following factors from part **(i)**
Use relevant method to find a constant
Obtain $A = -4$, following factors from part **(i)**
Obtain $B = 2$
Obtain $C = 5$

Q11:

(i) Substitute $x = 2$ and equate to zero, or divide by $x - 2$ and equate constant remainder to zero, or equivalent
Obtain $a = 4$

(ii) (a) Find further (quadratic or linear) factor by division, inspection or factor theorem or equivalent
Obtain $x^2 + 2x - 8$ or $x + 4$
State $(x - 2)^2(x + 4)$ or equivalent

(b) State any two of the four (or six) roots
State all roots ($\pm\sqrt{2}$, $\pm 2\mathrm{i}$), provided two are purely imaginary