Fayzan Munawar
Associate *of* Royal College of Science, UK
BSc Mathematics *(Imperial College London)*
tutoring@learningmathonline.com WhatsApp: +1 718 200 2476
Facebook/Learning Math Online

Algebra – Partial Fractions P3

Q1

(i) Express $\frac{10}{(2-x)(1+x^2)}$ in partial fractions. [5]

(ii) Hence, given that $|x| < 1$, obtain the expansion of $\frac{10}{(2-x)(1+x^2)}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [5]

Q2

Let $f(x) = \frac{7x+4}{(2x+1)(x+1)^2}$.

Express $f(x)$ in partial fractions. [5]

Q3

(i) Express $\frac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in ascending powers of x, up to and including the term in x^2. [5]

Q4

Let $f(x) \equiv \frac{x^2+3x+3}{(x+1)(x+3)}$.

Express $f(x)$ in partial fractions. [5]

Q5

Express $\frac{100}{x^2(10-x)}$ in partial fractions. [4]

Q6

(i) Express $\frac{5x+3}{(x+1)^2(3x+2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{5x+3}{(x+1)^2(3x+2)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

Q7

(i) Express $\frac{1+x}{(1-x)(2+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{1+x}{(1-x)(2+x^2)}$ in ascending powers of x, up to and including the term in x^2. [5]

Fayzan Munawar
Associate *of* Royal College of Science, UK
BSc Mathematics *(Imperial College London)*
tutoring@learningmathonline.com WhatsApp: +1 718 200 2476
Facebook/Learning Math Online

Q8

Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 1}{x^2(2x-1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x-1}.$$ [5]

Q9

(i) Express $\dfrac{4 + 5x - x^2}{(1-2x)(2+x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4 + 5x - x^2}{(1-2x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

Q10

Let $\mathrm{f}(x) = \dfrac{3x}{(1+x)(1+2x^2)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^3. [5]

Q11

(i) Express $\dfrac{5x - x^2}{(1+x)(2+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{5x - x^2}{(1+x)(2+x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

Q12

Let $\mathrm{f}(x) = \dfrac{12 + 8x - x^2}{(2-x)(4+x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2-x} + \dfrac{Bx + C}{4+x^2}$. [4]

Q13

By first expressing $\dfrac{4x^2 + 5x + 3}{2x^2 + 5x + 2}$ in partial fractions, show that

$$\int_0^4 \frac{4x^2 + 5x + 3}{2x^2 + 5x + 2}\,\mathrm{d}x = 8 - \ln 9.$$ [10]

Fayzan Munawar
Associate *of* Royal College of Science, UK
BSc Mathematics *(Imperial College London)*
tutoring@learningmathonline.com WhatsApp: +1 718 200 2476
Facebook/Learning Math Online

Q14

Let $f(x) = \dfrac{4x^2 - 7x - 1}{(x+1)(2x-3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\displaystyle\int_2^6 f(x)\,dx = 8 - \ln\left(\tfrac{49}{3}\right)$. [5]

Q15

(i) Express $\dfrac{9 - 7x + 8x^2}{(3-x)(1+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{9 - 7x + 8x^2}{(3-x)(1+x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

Fayzan Munawar
Associate *of* Royal College of Science, UK
BSc Mathematics *(Imperial College London)*
tutoring@learningmathonline.com WhatsApp: +1 718 200 2476
Facebook/Learning Math Online

Answers:

Q1:

(i) State or imply partial fractions are of the form $\frac{A}{2-x}+\frac{Bx+C}{1+x^2}$

Use any relevant method to obtain a constant
Obtain one of the values $A=2$, $B=2$, $C=4$
Obtain a second value
Obtain the third value

(ii) Use correct method to obtain the first two terms of the expansion of $(2-x)^{-1}$ or $(1-\frac{1}{2}x)^{-1}$ or $(1+x^2)^{-1}$

Obtain any correct unsimplified expansion of the partial fractions up to the terms in x^3, e.g. $(2x+4)(1+(-1)x^2)$ (deduct A1 for each incorrect expansion)

Carry out multiplication of expansion of $(1+x^2)^{-1}$ by $(2x+4)$

Obtain answer $5+\frac{5}{2}x-\frac{15}{4}x^2-\frac{15}{8}x^3$

Q2:

State or imply $f(x) \equiv \frac{A}{2x+1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$

Use any relevant method to obtain a constant
Obtain one of the values $A=2$, $B=-1$, $C=3$
Obtain the remaining two values

[A correct solution starting with third term $\frac{Cx}{(x+1)^2}$ or $\frac{Cx+D}{(x+1)^2}$ is also possible.]

Q3:

(i) State or imply the form $\frac{A}{1-x}+\frac{B}{1+2x}+\frac{C}{2+x}$	B1
Use any relevant method to determine a constant	M1
Obtain $A=1$, $B=2$ and $C=-4$	A1 + A1 + A1
(ii) Use correct method to obtain the first two terms of the expansion of $(1-x)^{-1}$, $(1+2x)^{-1}$, $(2+x)^{-1}$, or $(1+\frac{1}{2}x)^{-1}$	M1
Obtain complete unsimplified expansions up to x^2 of each partial fraction	A1√ + A1√ + A1√
Combine expansions and obtain answer $1-2x+\frac{17}{2}x^2$	A1

[Binomial coefficients such as $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient for the M1. The f.t. is on A, B, C.]

Q4:

State or imply the form $A+\frac{B}{x+1}+\frac{C}{x+3}$
State or obtain $A=1$
Use correct method for finding B or C
Obtain $B=\frac{1}{2}$
Obtain $C=-\frac{3}{2}$

Q5:

State or imply the form $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{10-x}$
Use any relevant method to determine a constant
Obtain one of the values $A=1$, $B=10$, $C=1$
Obtain the remaining two values
[The form $\frac{Dx+E}{x^2}+\frac{C}{10-x}$ is acceptable and leads to $D=1$, $E=10$, $C=1$]

Q6:

(i) State or imply partial fractions are of the form $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{3x+2}$	B1
Use any relevant method to obtain a constant	M1
Obtain one of the values $A=1$, $B=2$, $C=-3$	A1
Obtain a second value	A1
Obtain the third value	A1
(ii) Use correct method to obtain the first two terms of the expansion of $(x+1)^{-1}$, $(x+1)^{-2}$, $(3x+2)^{-1}$ or $(1+\frac{3}{2}x)^{-1}$	M1
Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction	A1√ + A1√ + A1√
Obtain answer $\frac{3}{2}-\frac{11}{4}x+\frac{29}{8}x^2$, or equivalent	A1

Fayzan Munawar
Associate *of* Royal College of Science, UK
BSc Mathematics *(Imperial College London)*
tutoring@learningmathonline.com WhatsApp: +1 718 200 2476
Facebook/Learning Math Online

Q7:

(i) State or imply partial fractions are of the form $\frac{A}{1-x}+\frac{Bx+C}{2+x^2}$

Use a relevant method to determine a constant

Obtain $A=\frac{2}{3}$, $B=\frac{2}{3}$ and $C=\frac{1}{3}$

(ii) Use correct method to find first two terms of the expansion of $(1-x)^{-1}$, $(2+x^2)^{-1}$ or $(1+\frac{1}{2}x^2)^{-1}$ M1

Obtain complete unsimplified expansions up to x^2 of each partial fraction e.g. $\frac{2}{3}(1+x+x^2)$ and $\frac{1}{2}(\frac{2}{3}x-\frac{1}{3})(1-\frac{1}{2}x^2)$ A1√ + A1√

Carry out multiplication of $(2+x^2)^{-1}$ by $(\frac{2}{3}x-\frac{1}{3})$, or equivalent, provided $BC \neq 0$ M1

Obtain answer $\frac{1}{2}+x+\frac{3}{4}x^2$ A1

Q8:

Divide by denominator and obtain quadratic remainder

Obtain $A = 1$

Use any relevant method to obtain B, C or D

Obtain one correct answer

Obtain $B = 2$, $C = 1$ and $D = -3$

Q9:

(i) State or imply partial fractions of the form $\frac{A}{1-2x}+\frac{B}{2+x}+\frac{C}{(2+x)^2}$ B1

Use any relevant method to determine a constant M1

Obtain one of the values $A = 1$, $B = 1$, $C = -2$ A1

Obtain a second value A1

Obtain the third value A1

[The form $\frac{A}{1-2x}+\frac{Dx+E}{(2+x)^2}$, where $A = 1$, $D = 1$, $E = 0$, is acceptable scoring B1M1A1A1A1 as above.]

(ii) Use correct method to obtain the first two terms of the expansion of $(1-2x)^{-1}$, $(2+x)^{-1}$, $(2+x)^{-2}$, $(1+\frac{1}{2}x)^{-1}$, or $(1+\frac{1}{2}x)^{-2}$ M1

Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction A1√ + A1√ + A1√

Obtain answer $1+\frac{9}{4}x+\frac{15}{4}x^2$, or equivalent A1

Q10:

(i) State or imply the form $\frac{A}{1+x}+\frac{Bx+C}{1+2x^2}$

Use any relevant method to evaluate a constant

Obtain one of $A = -1$, $B = 2$, $C = 1$

Obtain a second value

Obtain the third value

(ii) Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$ or $(1+2x^2)^{-1}$

Obtain correct expansion of each partial fraction as far as necessary

Multiply out fully by $Bx + C$, where $BC \neq 0$

Obtain answer $3x - 3x^2 - 3x^3$

Q11:

(i) State or imply partial fractions are of the form $\frac{A}{1+x}+\frac{Bx+C}{2+x^2}$ B1

Use a relevant method to determine a constant M1

Obtain one of the values $A = -2$, $B = 1$, $C = 4$ A1

Obtain a second value A1

Obtain the third value A1

(ii) Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$, $\left(1+\frac{1}{2}x^2\right)^{-1}$ or $(2+x^2)^{-1}$ in ascending powers of x M1

Obtain correct unsimplified expansion up to the term in x^3 of each partial fraction A1√ + A1√

Multiply out fully by $Bx + C$, where $BC \neq 0$ M1

Obtain final answer $\frac{5}{2}x-3x^2+\frac{7}{4}x^3$, or equivalent A1

Q12:

Use any relevant method to determine a constant

Obtain one of the values $A = 3$, $B = 4$, $C = 0$

Obtain a second value

Obtain the third value

Fayzan Munawar
Associate *of* Royal College of Science, UK
BSc Mathematics *(Imperial College London)*
tutoring@learningmathonline.com WhatsApp: +1 718 200 2476
Facebook/Learning Math Online

Q13:

State or imply form $A+\frac{B}{2x+1}+\frac{C}{x+2}$

State or obtain $A=2$

Use correct method for finding B or C

Obtain $B=1$

Obtain $C=-3$

Obtain $2x+\frac{1}{2}\ln(2x+1)-3\ln(x+2)$ [Deduct B1√ for each error or omission]

Substitute limits in expression containing $a\ln(2x+1)+b\ln(x+2)$

Show full and exact working to confirm that $8+\frac{1}{2}\ln 9-3\ln 6+3\ln 2$, or an equivalent expression, simplifies to given result $8-\ln 9$

Q14:

(i) State or imply the form $A+\frac{B}{x+1}+\frac{C}{2x-3}$

State or obtain $A=2$

Use a correct method for finding a constant

Obtain $B=-2$

Obtain $C=-1$

(ii) Obtain integral $2x-2\ln(x+1)-\frac{1}{2}\ln(2x-3)$

(Deduct B1√ for each error or omission. The f.t. is on A, B, C.)

Substitute limits correctly in an expression containing terms $a\ln(x+1)$ and $b\ln(2x-3)$

Obtain the given answer following full and exact working

[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in **(i)**; B1√B1√M1A0 in **(ii)**.]

[SR: For a solution starting with $\frac{B}{x+1}+\frac{Dx+E}{2x-3}$, give M1A1 for one of $B=-2$, $D=4$, $E=-7$ and A1 for the other two constants; then give B1B1 for $A=2$, $C=-1$.]

[SR: For a solution starting with $\frac{Fx+G}{x+1}+\frac{C}{2x-3}$ or with $\frac{Fx}{x+1}+\frac{C}{2x-3}$, give M1A1 for one of $C=-1$, $F=2$, $G=0$ and A1 for the other constants or constant; then give B1B1 for $A=2$, $B=-2$.]

Q15:

(i) State or imply form $\frac{A}{3-x}+\frac{Bx+C}{1+x^2}$

Use relevant method to determine a constant

Obtain $A=6$

Obtain $B=-2$

Obtain $C=1$

(ii) Either Use correct method to obtain first two terms of expansion of $(3-x)^{-1}$ or $\left(1-\frac{1}{3}x\right)^{-1}$ or $(1+x^2)^{-1}$

Obtain $\frac{A}{3}\left(1+\frac{1}{3}x+\frac{1}{9}x^2+\frac{1}{27}x^3\right)$

Obtain $(Bx+C)(1-x^2)$

Obtain sufficient terms of the product $(Bx+C)(1-x^2)$, $B,C\neq 0$ and add the two expansions

Obtain final answer $3-\frac{4}{3}x-\frac{7}{9}x^2+\frac{56}{27}x^3$