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Numerical Solutions – Iteration P3

Q1

(i) By sketching a suitable pair of graphs, show that the equation

$$2\cot x = 1 + e^{x},$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1.0. [2]

(iii) Show that this root also satisfies the equation

$$x = \tan^{-1}\left(\frac{2}{1 + e^{x}}\right).$$ [1]

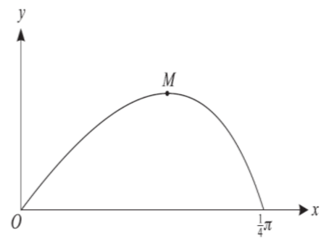
(iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{2}{1 + e^{x_n}}\right),$$

with initial value $x_1 = 0.7$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q2

The diagram shows the curve $y = x\cos 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The point M is a maximum point.



(i) Show that the x-coordinate of M satisfies the equation $1 = 2x\tan 2x$. [3]

(ii) The equation in part **(i)** can be rearranged in the form $x = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x}\right)$. Use the iterative formula

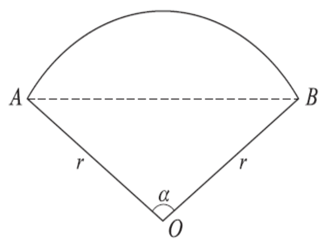
$$x_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x_n}\right),$$

with initial value $x_1 = 0.4$, to calculate the x-coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(iii) Use integration by parts to find the exact area of the region enclosed between the curve and the x-axis from 0 to $\frac{1}{4}\pi$. [5]

Q3

The diagram shows a sector AOB of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \pi$. The area of triangle AOB is half the area of the sector.



(i) Show that α satisfies the equation

$$x = 2\sin x.$$ [2]

(ii) Verify by calculation that α lies between $\frac{1}{2}\pi$ and $\frac{2}{3}\pi$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{3}(x_n + 4\sin x_n)$$

converges, then it converges to a root of the equation in part **(i)**. [2]

(iv) Use this iterative formula, with initial value $x_1 = 1.8$, to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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Q4

(i) By sketching a suitable pair of graphs, show that the equation

$$2 - x = \ln x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between 1.4 and 1.7. [2]

(iii) Show that this root also satisfies the equation

$$x = \tfrac{1}{3}(4 + x - 2\ln x).$$ [1]

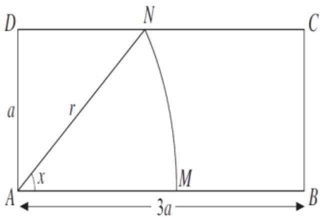
(iv) Use the iterative formula

$$x_{n+1} = \tfrac{1}{3}(4 + x_n - 2\ln x_n),$$

with initial value $x_1 = 1.5$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q5

In the diagram, $ABCD$ is a rectangle with $AB = 3a$ and $AD = a$. A circular arc, with centre A and radius r, joins points M and N on AB and CD respectively. The angle MAN is x radians. The perimeter of the sector AMN is equal to half the perimeter of the rectangle.



(i) Show that x satisfies the equation

$$\sin x = \tfrac{1}{4}(2 + x).$$ [3]

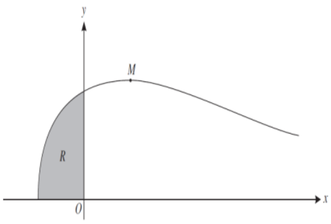
(ii) This equation has only one root in the interval $0 < x < \frac{1}{2}\pi$. Use the iterative formula

$$x_{n+1} = \sin^{-1}\left(\frac{2 + x_n}{4}\right),$$

with initial value $x_1 = 0.8$, to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q6

The diagram shows the curve $y = e^{-\frac{1}{2}x}\sqrt{(1 + 2x)}$ and its maximum point M. The shaded region between the curve and the axes is denoted by R.



(i) Find the x-coordinate of M. [4]

(ii) Find by integration the volume of the solid obtained when R is rotated completely about the x-axis. Give your answer in terms of π and e. [6]

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Q7

The constant a is such that $\int_0^a x e^{\frac{1}{2}x}\, dx = 6$.

(i) Show that a satisfies the equation

$$x = 2 + e^{-\frac{1}{2}x}.$$ [5]

(ii) By sketching a suitable pair of graphs, show that this equation has only one root. [2]

(iii) Verify by calculation that this root lies between 2 and 2.5. [2]

(iv) Use an iterative formula based on the equation in part **(i)** to calculate the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q8

The equation $x^3 - 2x - 2 = 0$ has one real root.

(i) Show by calculation that this root lies between $x = 1$ and $x = 2$. [2]

(ii) Prove that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2 - 2}$$

converges, then it converges to this root. [2]

(iii) Use this iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q9

The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{15}{x_n^3},$$

with initial value $x_1 = 3$, converges to α.

(i) Use this iterative formula to find α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

(ii) State an equation satisfied by α and hence find the exact value of α. [2]

Q10

The equation $x^3 - 8x - 13 = 0$ has one real root.

(i) Find the two consecutive integers between which this root lies. [2]

(ii) Use the iterative formula

$$x_{n+1} = (8x_n + 13)^{\frac{1}{3}}$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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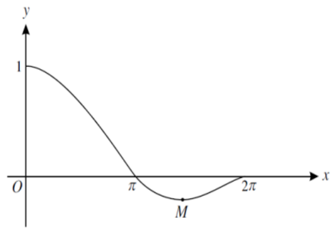
Q11



The diagram shows the curve $y = \frac{\sin x}{x}$ for $0 < x \leqslant 2\pi$, and its minimum point M.

(i) Show that the x-coordinate of M satisfies the equation

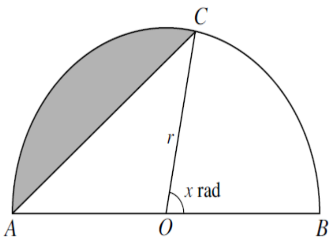
$$x = \tan x. \quad [4]$$

(ii) The iterative formula

$$x_{n+1} = \tan^{-1}(x_n) + \pi$$

can be used to determine the x-coordinate of M. Use this formula to determine the x-coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q12



The diagram shows a semicircle ACB with centre O and radius r. The angle BOC is x radians. The area of the shaded segment is a quarter of the area of the semicircle.

(i) Show that x satisfies the equation

$$x = \tfrac{3}{4}\pi - \sin x. \quad [3]$$

(ii) This equation has one root. Verify by calculation that the root lies between 1.3 and 1.5. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{3}{4}\pi - \sin x_n$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q13

The curve $y = \frac{\ln x}{x + 1}$ has one stationary point.

(i) Show that the x-coordinate of this point satisfies the equation

$$x = \frac{x + 1}{\ln x},$$

and that this x-coordinate lies between 3 and 4. [5]

(ii) Use the iterative formula

$$x_{n+1} = \frac{x_n + 1}{\ln x_n}$$

to determine the x-coordinate correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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Q14

(i) By sketching suitable graphs, show that the equation

$$4x^2 - 1 = \cot x$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.6 and 1. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{1}{2}\sqrt{(1 + \cot x_n)}$$

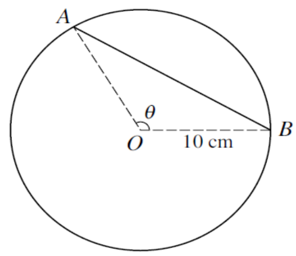
to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q15

(i) Given that $\displaystyle\int_1^a \frac{\ln x}{x^2}\,dx = \frac{2}{5}$, show that $a = \frac{5}{3}(1 + \ln a)$. [5]

(ii) Use an iteration formula based on the equation $a = \frac{5}{3}(1 + \ln a)$ to find the value of a correct to 2 decimal places. Use an initial value of 4 and give the result of each iteration to 4 decimal places. [3]

Q16

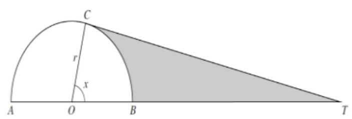


The diagram shows a circle with centre O and radius 10 cm. The chord AB divides the circle into two regions whose areas are in the ratio 1 : 4 and it is required to find the length of AB. The angle AOB is θ radians.

(i) Show that $\theta = \frac{2}{5}\pi + \sin\theta$. [3]

(ii) Showing all your working, use an iterative formula, based on the equation in part **(i)**, with an initial value of 2.1, to find θ correct to 2 decimal places. Hence find the length of AB in centimetres correct to 1 decimal place. [5]

Q17



The diagram shows a semicircle ACB with centre O and radius r. The tangent at C meets AB produced at T. The angle BOC is x radians. The area of the shaded region is equal to the area of the semicircle.

(i) Show that x satisfies the equation

$$\tan x = x + \pi.$$ [3]

(ii) Use the iterative formula $x_{n+1} = \tan^{-1}(x_n + \pi)$ to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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Q18

(i) By sketching a suitable pair of graphs, show that the equation

$$\cot x = 1 + x^2,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 0.8. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{1 + x_n^2}\right)$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q19

(i) By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - \tfrac{1}{2}x^2,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1 and 1.4. [2]

(iii) Show that this root also satisfies the equation

$$x = \cos^{-1}\left(\frac{2}{6 - x^2}\right).$$ [1]

(iv) Use an iterative formula based on the equation in part **(iii)** to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q20

It is given that $\int_1^a x \ln x \, dx = 22$, where a is a constant greater than 1.

(i) Show that $a = \sqrt{\left(\frac{87}{2\ln a - 1}\right)}$. [5]

(ii) Use an iterative formula based on the equation in part **(i)** to find the value of a correct to 2 decimal places. Use an initial value of 6 and give the result of each iteration to 4 decimal places. [3]

Q21

(i) It is given that $2\tan 2x + 5\tan^2 x = 0$. Denoting $\tan x$ by t, form an equation in t and hence show that either $t = 0$ or $t = \sqrt[3]{(t + 0.8)}$. [4]

(ii) It is given that there is exactly one real value of t satisfying the equation $t = \sqrt[3]{(t + 0.8)}$. Verify by calculation that this value lies between 1.2 and 1.3. [2]

(iii) Use the iterative formula $t_{n+1} = \sqrt[3]{(t_n + 0.8)}$ to find the value of t correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iv) Using the values of t found in previous parts of the question, solve the equation

$$2\tan 2x + 5\tan^2 x = 0$$

for $-\pi \leqslant x \leqslant \pi$. [3]

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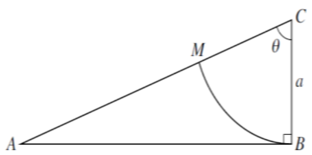
Q22

In the diagram, ABC is a triangle in which angle ABC is a right angle and $BC = a$. A circular arc, with centre C and radius a, joins B and the point M on AC. The angle ACB is θ radians. The area of the sector CMB is equal to one third of the area of the triangle ABC.



(i) Show that θ satisfies the equation

$$\tan\theta = 3\theta.$$ [2]

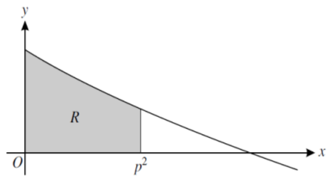
(ii) This equation has one root in the interval $0 < \theta < \frac{1}{2}\pi$. Use the iterative formula

$$\theta_{n+1} = \tan^{-1}(3\theta_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q23

The diagram shows part of the curve $y = \cos(\sqrt{x})$ for $x \geqslant 0$, where x is in radians. The shaded region between the curve, the axes and the line $x = p^2$, where $p > 0$, is denoted by R. The area of R is equal to 1.

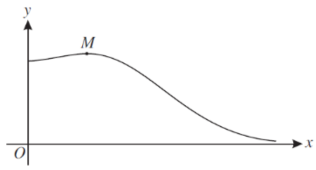


(i) Use the substitution $x = u^2$ to find $\int_0^{p^2} \cos(\sqrt{x})\,dx$. Hence show that $\sin p = \frac{3 - 2\cos p}{2p}$. [6]

(ii) Use the iterative formula $p_{n+1} = \sin^{-1}\left(\frac{3 - 2\cos p_n}{2p_n}\right)$, with initial value $p_1 = 1$, to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q24

The diagram shows the curve $y = e^{-\frac{1}{2}x^2}\sqrt{(1 + 2x^2)}$ for $x \geqslant 0$, and its maximum point M.



(i) Find the exact value of the x-coordinate of M. [4]

(ii) The sequence of values given by the iterative formula

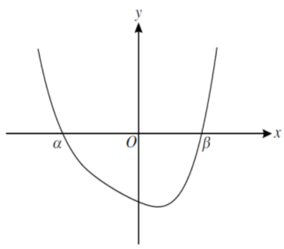
$$x_{n+1} = \sqrt{\left(\ln(4 + 8x_n^2)\right)},$$

with initial value $x_1 = 2$, converges to a certain value α. State an equation satisfied by α and hence show that α is the x-coordinate of a point on the curve where $y = 0.5$. [3]

(iii) Use the iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q25

The diagram shows the curve $y = x^4 + 2x^3 + 2x^2 - 4x - 16$, which crosses the x-axis at the points $(\alpha, 0)$ and $(\beta, 0)$ where $\alpha < \beta$. It is given that α is an integer.



(i) Find the value of α. [2]

(ii) Show that β satisfies the equation $x = \sqrt[3]{(8 - 2x)}$. [3]

(iii) Use an iteration process based on the equation in part **(ii)** to find the value of β correct to 2 decimal places. Show the result of each iteration to 4 decimal places. [3]

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Answers:

Q1:

(i) Make recognizable sketch of a relevant graph, e.g. $y = 2\cot x$
Sketch an appropriate second graph, e.g. $y = 1 + e^x$ correctly and justify the given statement

(ii) Consider sign of $2\cot x - 1 - e^x$ at $x = 0.5$ and $x = 1$, or equivalent
Complete the argument with appropriate calculations

(iii) Show that the given equation is equivalent to $x = \tan^{-1}\left(\frac{2}{1+e^x}\right)$, or vice versa

(iv) Use the iterative formula correctly at least once
Obtain final answer 0.61
Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (0.605, 0.615)

Q2:

(i) Use product rule
Obtain correct derivative $\cos 2x - 2x \sin 2x$
Equate derivative to zero and obtain given answer correctly

(ii) Use the iterative formula correctly at least once
Obtain final answer 0.43
Show sufficient iterations to at least 3d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.425, 0.435)

(iii) Attempt integration by parts and obtain $\pm kx \sin 2x \pm \int l \sin 2x \, dx$, where $k, l = \frac{1}{2}, 1$, or 2
Obtain $\frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$
Obtain indefinite integral $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x$
Use limits $x = 0$ and $x = \frac{1}{4}\pi$ having integrated twice
Obtain answer $\frac{1}{8}\pi - \frac{1}{4}$, or exact equivalent

Q3:

(i) Using the formulae $\frac{1}{2} r^2 \alpha$ and $\frac{1}{2} r^2 \sin \alpha$, or equivalent, form an equation
Obtain given equation correctly
[Allow the use of OA and/or OB for r.]

(ii) Consider sign of $x - 2 \sin x$ at $x = \frac{1}{2}\pi$ and $x = \frac{2}{3}\pi$, or equivalent
Complete the argument correctly with appropriate calculations

(iii) State or imply the equation $x = \frac{1}{3}(x + 4 \sin x)$
Rearrange this as $x = 2 \sin x$, or work *vice versa*

(iv) Use the iterative formula correctly at least once
Obtain final answer 1.90
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.895, 1.905)
[The final answer 1.9 scores A0.]

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Q4:

(i) Make a recognisable sketch of an appropriate graph, e.g. $y = \ln x$
Sketch an appropriate second graph, e.g. $y = 2 - x$, correctly and justify the given statement

(ii) Consider sign of $2 - x - \ln x$ when $x = 1.4$ and $x = 1.7$, or equivalent
Complete the argument with correct calculations

(iii) Rearrange the equation $x = \frac{1}{3}(4 + x - 2\ln x)$ as $2 - x = \ln x$, or *vice versa*

(iv) Use the iterative formula correctly at least once
Obtain final answer 1.56
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.555, 1.565)

Q5:

(i) State or imply $r = a \operatorname{cosec} x$, or equivalent
Using perimeters, obtain a correct equation in x, e.g. $2a \operatorname{cosec} x + ax \operatorname{cosec} x = 4a$, or $2r + rx = 4a$
Deduce the given form of equation correctly

(ii) Use the iterative formula correctly at least once
Obtain final answer 0.76
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show that there is a sign change in the value of $\sin x - \frac{1}{4}(2 + x)$ in the interval (0.755, 0.765)

Q6:

(i) Either use correct product or quotient rule, or square both sides, use correct product rule and make a reasonable attempt at applying the chain rule
Obtain correct result of differentiation in any form
Set derivative equal to zero and solve for x
Obtain $x = \frac{1}{2}$ only, correctly

(ii) State or imply the indefinite integral for the volume is $\pi \int e^{-x}(1 + 2x)\,dx$

Integrate by parts and reach $\pm e^{-x}(1 + 2x) \pm \int 2e^{-x}\,dx$

Obtain $-e^{-x}(1 + 2x) + \int 2e^{-x}\,dx$, or equivalent

Complete integration correctly, obtaining $-e^{-x}(1 + 2x) - 2e^{-x}$, or equivalent

Use limits $x = -\frac{1}{2}$ and $x = 0$ correctly, having integrated twice

Obtain exact answer $\pi(2\sqrt{e} - 3)$, or equivalent

[If π omitted initially or 2π or $\pi/2$ used, give B0 and then follow through.]

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Q7:

(i) Integrate by parts and reach $kxe^{\frac{1}{2}x} - k\int e^{\frac{1}{2}x}\,dx$

Obtain $2xe^{\frac{1}{2}x} - 2\int e^{\frac{1}{2}x}\,dx$

Complete the integration, obtaining $2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$, or equivalent
Substitute limits correctly and equate result to 6, having integrated twice
Rearrange and obtain $a = e^{-\frac{1}{2}a} + 2$

(ii) Make recognizable sketch of a relevant exponential graph, e.g. $y = e^{-\frac{1}{2}x} + 2$
Sketch a second relevant straight line graph, e.g. $y = x$, or curve, and indicate the root

(iii) Consider sign of $x - e^{-\frac{1}{2}x} - 2$ at $x = 2$ and $x = 2.5$, or equivalent
Justify the given statement with correct calculations and argument

(iv) Use the iterative formula $x_{n+1} = 2 + e^{-\frac{1}{2}x_n}$ correctly at least once, with $2 \le x_n \le 2.5$
Obtain final answer 2.31
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (2.305, 2.315)

Q8:

(i) Compare signs of $x^3 - 2x - 2$ when $x = 1$ and $x = 2$, or equivalent
Complete the argument with correct calculations

(ii) State or imply the equation $x = (2x^3 + 2) / (3x^2 - 2)$
Rearrange this in the form $x^3 - 2x - 2 = 0$, or work *vice versa*

(iii) Use the iterative formula correctly at least once with $x_n > 0$
Obtain final answer 1.77
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p.,
or show there is a sign change
In the interval (1.765, 1.775)

Q9:

(i) Use the iterative formula correctly at least once
State final answer 2.78
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in an appropriate function in (2.775, 2.785)

(ii) State a suitable equation, e.g. $x = \frac{3}{4}x + \frac{15}{x^3}$

State that the exact value of α is $\sqrt[4]{60}$, or equivalent

Q10:

(i) Evaluate, or consider the sign of, $x^3 - 8x - 13$ for two integer values of x, or equivalent
Conclude $x = 3$ and $x = 4$ with no errors seen

(ii) Use the iterative formula correctly at least once
Obtain final answer 3.43
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (3.425, 3.435)

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Q11:

(i) Use correct quotient or product rule
Obtain correct derivative in any form
Equate derivative to zero and solve for x
Obtain the given answer correctly

(ii) Use the iterative formula correctly at least once
Obtain final answer 4.49
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show that there is a sign change in the interval (4.485, 4.495)

Q12:

(i) Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}r^2\sin\theta$, or equivalent, form an equation
Obtain a correct equation in r and x and/or $x/2$ in any form
Obtain the given equation correctly

(ii) Consider the sign of $x-(\frac{3}{4}\pi-\sin x)$ at $x = 1.3$ and $x = 1.5$, or equivalent
Complete the argument with correct calculations

(iii) Use the iterative formula correctly at least once
Obtain final answer 1.38
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.375, 1.385)

Q13:

(i) Use correct quotient or product rule
Obtain correct derivative in any form, e.g. $\dfrac{1}{x(x+1)}-\dfrac{\ln x}{(x+1)^2}$
Equate derivative to zero and obtain the given equation correctly
Consider the sign of $x-\dfrac{(x+1)}{\ln x}$ at $x = 3$ and $x = 4$, or equivalent
Complete the argument with correct calculated values

(ii) Use the iterative formula correctly at least once, using or reaching a value in the interval (3, 4)
Obtain final answer 3.59
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (3.585, 3.595)

Q14:

(i) Make recognisable sketch of a relevant graph over the given range
Sketch the other relevant graph on the same diagram and justify the given statement

(ii) Consider sign of $4x^2 - 1 - \cot x$ at $x = 0.6$ and $x = 1$, or equivalent
Complete the argument correctly with correct calculated values

(iii) Use the iterative formula correctly at least once
Obtain final answer 0.73
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.725, 0.735)

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Q15:

(i) Attempt integration by parts

Obtain $-x^{-1}\ln x + \int \frac{1}{x^2}\,dx$, $\frac{x\ln x - x}{x^2} + 2\int \frac{\ln x}{x^2}\,dx - 2\int \frac{1}{x^2}\,dx$ or equivalent

Obtain $-x^{-1}\ln x - x^{-1}$ or equivalent

Use limits correctly, equate to $\frac{2}{5}$ and attempt rearrangement to obtain a in terms of $\ln a$

Obtain given answer $a = \frac{5}{3}(1+\ln a)$ correctly

(ii) Use valid iterative formula correctly at least once

Obtain final answer 3.96

Show sufficient iterations to > 4 dp to justify accuracy to 2 dp or show sign change in interval (3.955, 3.965)

[4 → 3.9772 → 3.9676 → 3.9636 → 3.9619]

SR: Use of $a_{n+1} = e^{\left(\frac{3}{5}a_n - 1\right)}$ to obtain 0.50 also earns 3/3.

Q16:

(i) State or imply area of segment is $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ or $50\theta - 50\sin\theta$

Attempt to form equation from area of segment = $\frac{1}{5}$ of area of circle, or equivalent

Confirm given result $\theta = \frac{2}{5}\pi + \sin\theta$

(ii) Use iterative formula correctly at least once

Obtain value for θ of 2.11

Show sufficient iterations to justify value of θ or show sign change in interval (2.105, 2.115)

Use correct trigonometry to find an expression for the length of AB

e.g. $20\sin 1.055$ or $\sqrt{200 - 200\cos 2.11}$

Hence 17.4

[2.1 → 2.1198 → 2.1097 → 2.1149 → 2.1122]

Q17:

(i) State or imply $CT = r\tan x$ or $OT = r\sec x$, or equivalent

Using correct area formulae, form an equation in r and x

Obtain the given answer correctly

(ii) Use the iterative formula correctly at least once

Obtain the final answer 1.35

Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p. , or show there is a sign change in the interval (1.345, 1.355)

Q18:

(i) Make recognisable sketch of a relevant graph over the given range

Sketch the other relevant graph and justify the given statement

(ii) Consider the sign of $\cot x - (1 + x^2)$ at $x = 0.5$ and $x = 0.8$, or equivalent

Complete the argument with correct calculated values

(iii) Use the iterative formula correctly at least once with $0.5 \le x_n \le 0.8$

Obtain final answer 0.62

Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.615, 0.625)

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Q19:

(i) Make recognisable sketch of a relevant graph over the given interval
Sketch the other relevant graph and justify the given statement

(ii) Consider the sign of $\sec x - (3 - \frac{1}{2}x^2)$ at $x = 1$ and $x = 1.4$, or equivalent
Complete the argument with correct calculated values

(iii) Convert the given equation to $\sec x = 3 - \frac{1}{2}x^2$ or work *vice versa*

(iv) Use a correct iterative formula correctly at least once
Obtain final answer 1.13
Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change in the interval (1.125, 1.135)
[SR: Successive evaluation of the iterative function with $x = 1, 2, \ldots$ scores M0.]

Q20:

(i) **Either**
Use integration by parts and reach an expression $kx^2 \ln x \pm n \int x^2 . \frac{1}{x} dx$

Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$ or equivalent

Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$

(ii) Use iterative process correctly at least once

Obtain final answer 5.86

Show sufficient iterations to 4 d.p. to justify 5.86 or show a sign change in the interval (5.855, 5.865)

$(6 \to 5.8030 \to 5.8795 \to 5.8491 \to 5.8611 \to 5.8564)$

Q21:

(i) Use correct identity for $\tan 2x$ and obtains $at^4 + bt^3 + ct^2 + dt = 0$, where b may be zero
Obtain correct horizontal equation, e.g. $4t + 5t^2 - 5t^4 = 0$
Obtain $kt(t^3 + et + f) = 0$ or equivalent
Confirm given results $t = 0$ and $t = \sqrt[3]{t + 0.8}$

(ii) Consider sign of $t - \sqrt[3]{t + 0.8}$ at 1.2 and 1.3 or equivalent
Justify the given statement with correct calculations (–0.06 and 0.02)

(iii) Use the iterative formula correctly at least once with $1.2 < t_n < 1.3$
Obtain final answer 1.276
Show sufficient iterations to justify answer or show there is a change of sign in interval (1.2755, 1.2765)

(iv) Evaluate $\tan^{-1}$ (answer from part **(iii)**) to obtain at least one value
Obtain –2.24 and 0.906
State $-\pi$, 0 and π
[SR If A0, B0, allow B1 for any 3 roots]

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Q22:

(i) Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}bh$, form an equation an a and θ
Obtain given answer

(ii) Use the iterative formula correctly at least once
Obtain answer $\theta = 1.32$
Show sufficient iterations to 4 d.p. to justify 1.32 to 2 d.p., or show there is a sign change in the interval (1.315, 1.325)

Q23:

(i) Substitute for x and dx throughout the integral
Obtain $\int 2u \cos u \, du$
Integrate by parts and obtain answer of the form $au \sin u + b \cos u$, where $ab \neq 0$
Obtain $2u \sin u + 2\cos u$
Use limits $u = 0$, $u = p$ correctly and equate result to 1
Obtain the given answer

(ii) Use the iterative formula correctly at least once
Obtain final answer $p = 1.25$
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.245, 1.255)

Q24:

(i) Use correct product or quotient rule and use chain rule at least once
Obtain derivative in any correct form
Equate derivative to zero and solve an equation with at least two non-zero terms for real x
Obtain answer $x = \frac{1}{\sqrt{2}}$, or exact equivalent

(ii) State a suitable equation, e.g. $\alpha = \sqrt{\ln(4 + 8\alpha^2)}$
Rearrange to reach $e^{\alpha^2} = 4 + 8\alpha^2$
Obtain $\frac{1}{2} = e^{-\frac{1}{2}\alpha^2}\sqrt{1 + 2\alpha^2}$, or work *vice versa*

(iii) Use the iterative formula correctly at least once
Obtain final answer 1.86
Show sufficient iterations to 4 d.p. to justify 1.86 to 2 d.p., or show there is a sign change in the interval (1.855, 1.865)

Q25:

(i) Find y for $x = -2$
Obtain 0 and conclude that $\alpha = -2$

(ii) Either Find cubic factor by division or inspection or equivalent
Obtain $x^3 + 2x - 8$
Rearrange to confirm given equation $x = \sqrt[3]{8 - 2x}$

(iii) Use the given iterative formula correctly at least once
Obtain final answer 1.67
Show sufficient iterations to at least 4 d.p. to justify answer 1.67 to 2 d.p. or show there is a change of sign in interval (1.665, 1.675)