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Functions P1

Q1

The function f is defined by $f : x \mapsto ax + b$, for $x \in \mathbb{R}$, where a and b are constants. It is given that $f(2) = 1$ and $f(5) = 7$.

(i) Find the values of a and b. [2]

(ii) Solve the equation $ff(x) = 0$. [3]

Q2

The equation of a curve is $y = 8x - x^2$.

(i) Express $8x - x^2$ in the form $a - (x + b)^2$, stating the numerical values of a and b. [3]

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve. [2]

(iii) Find the set of values of x for which $y \geqslant -20$. [3]

The function g is defined by $g : x \mapsto 8x - x^2$, for $x \geqslant 4$.

(iv) State the domain and range of g^{-1}. [2]

(v) Find an expression, in terms of x, for $g^{-1}(x)$. [3]

Q3

The functions f and g are defined as follows:

$$f : x \mapsto x^2 - 2x, \quad x \in \mathbb{R},$$
$$g : x \mapsto 2x + 3, \quad x \in \mathbb{R}.$$

(i) Find the set of values of x for which $f(x) > 15$. [3]

(ii) Find the range of f and state, with a reason, whether f has an inverse. [4]

(iii) Show that the equation $gf(x) = 0$ has no real solutions. [3]

(iv) Sketch, in a single diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$, making clear the relationship between the graphs. [2]

Q4

Functions f and g are defined by

$$f : x \mapsto 2x - 5, \quad x \in \mathbb{R},$$
$$g : x \mapsto \frac{4}{2 - x}, \quad x \in \mathbb{R}, \quad x \neq 2.$$

(i) Find the value of x for which $fg(x) = 7$. [3]

(ii) Express each of $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x. [3]

(iii) Show that the equation $f^{-1}(x) = g^{-1}(x)$ has no real roots. [3]

(iv) Sketch, on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between these two graphs. [3]

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Q5

The function $f : x \mapsto 5\sin^2 x + 3\cos^2 x$ is defined for the domain $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a + b\sin^2 x$, stating the values of a and b. [2]

(ii) Hence find the values of x for which $f(x) = 7\sin x$. [3]

(iii) State the range of f. [2]

Q6

The function $f : x \mapsto 2x - a$, where a is a constant, is defined for all real x.

(i) In the case where $a = 3$, solve the equation $ff(x) = 11$. [3]

The function $g : x \mapsto x^2 - 6x$ is defined for all real x.

(ii) Find the value of a for which the equation $f(x) = g(x)$ has exactly one real solution. [3]

The function $h : x \mapsto x^2 - 6x$ is defined for the domain $x \geqslant 3$.

(iii) Express $x^2 - 6x$ in the form $(x - p)^2 - q$, where p and q are constants. [2]

(iv) Find an expression for $h^{-1}(x)$ and state the domain of h^{-1}. [4]

Q7

A function f is defined by $f : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant 360°$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

A function g is defined by $g : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant A°$, where A is a constant.

(iii) State the largest value of A for which g has an inverse. [1]

(iv) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$. [2]

Q8

A function f is defined by $f : x \mapsto (2x - 3)^3 - 8$, for $2 \leqslant x \leqslant 4$.

(i) Find an expression, in terms of x, for $f'(x)$ and show that f is an increasing function. [4]

(ii) Find an expression, in terms of x, for $f^{-1}(x)$ and find the domain of f^{-1}. [4]

Q9

Functions f and g are defined by

$$f : x \mapsto k - x \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$
$$g : x \mapsto \frac{9}{x + 2} \quad \text{for } x \in \mathbb{R},\ x \neq -2.$$

(i) Find the values of k for which the equation $f(x) = g(x)$ has two equal roots and solve the equation $f(x) = g(x)$ in these cases. [6]

(ii) Solve the equation $fg(x) = 5$ when $k = 6$. [3]

(iii) Express $g^{-1}(x)$ in terms of x. [2]

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Q10

The function f is defined by $f : x \mapsto x^2 - 3x$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $f(x) > 4$. [3]

(ii) Express $f(x)$ in the form $(x - a)^2 - b$, stating the values of a and b. [2]

(iii) Write down the range of f. [1]

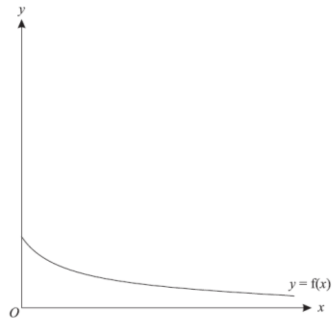
(iv) State, with a reason, whether f has an inverse. [1]

The function g is defined by $g : x \mapsto x - 3\sqrt{x}$ for $x \geqslant 0$.

(v) Solve the equation $g(x) = 10$. [3]

Q11

The diagram shows the graph of $y = f(x)$, where $f : x \mapsto \frac{6}{2x + 3}$ for $x \geqslant 0$.



(i) Find an expression, in terms of x, for $f'(x)$ and explain how your answer shows that f is a decreasing function. [3]

(ii) Find an expression, in terms of x, for $f^{-1}(x)$ and find the domain of f^{-1}. [4]

(iii) Copy the diagram and, on your copy, sketch the graph of $y = f^{-1}(x)$, making clear the relationship between the graphs. [2]

The function g is defined by $g : x \mapsto \frac{1}{2}x$ for $x \geqslant 0$.

(iv) Solve the equation $fg(x) = \frac{3}{2}$. [3]

Q12

The function f is defined by $f : x \mapsto 2x^2 - 8x + 11$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) Explain why f does not have an inverse. [1]

The function g is defined by $g : x \mapsto 2x^2 - 8x + 11$ for $x \leqslant A$, where A is a constant.

(iv) State the largest value of A for which g has an inverse. [1]

(v) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$ and state the range of g^{-1}. [4]

Q13

The function f is such that $f(x) = (3x + 2)^3 - 5$ for $x \geqslant 0$.

(i) Obtain an expression for $f'(x)$ and hence explain why f is an increasing function. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

Q14

Functions f and g are defined by

$$f : x \mapsto 4x - 2k \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$g : x \mapsto \frac{9}{2 - x} \quad \text{for } x \in \mathbb{R}, x \neq 2.$$

(i) Find the values of k for which the equation $fg(x) = x$ has two equal roots. [4]

(ii) Determine the roots of the equation $fg(x) = x$ for the values of k found in part **(i)**. [3]

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Q15

The function f is defined by

$$f : x \mapsto 3x - 2 \quad \text{for } x \in \mathbb{R}.$$

(i) Sketch, in a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the two graphs. [2]

The function g is defined by

$$g : x \mapsto 6x - x^2 \quad \text{for } x \in \mathbb{R}.$$

(ii) Express $gf(x)$ in terms of x, and hence show that the maximum value of $gf(x)$ is 9. [5]

The function h is defined by

$$h : x \mapsto 6x - x^2 \quad \text{for } x \geqslant 3.$$

(iii) Express $6x - x^2$ in the form $a - (x - b)^2$, where a and b are positive constants. [2]

(iv) Express $h^{-1}(x)$ in terms of x. [3]

Q16

The function f is defined by $f : x \mapsto 2x^2 - 12x + 13$ for $0 \leqslant x \leqslant A$, where A is a constant.

(i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the value of A for which the graph of $y = f(x)$ has a line of symmetry. [1]

(iii) When A has this value, find the range of f. [2]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 13$ for $x \geqslant 4$.

(iv) Explain why g has an inverse. [1]

(v) Obtain an expression, in terms of x, for $g^{-1}(x)$. [3]

Q17

Functions f and g are defined by

$$f : x \mapsto 2x + 1, \quad x \in \mathbb{R}, \ x > 0,$$
$$g : x \mapsto \frac{2x - 1}{x + 3}, \quad x \in \mathbb{R}, \ x \neq -3.$$

(i) Solve the equation $gf(x) = x$. [3]

(ii) Express $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x. [4]

(iii) Show that the equation $g^{-1}(x) = x$ has no solutions. [3]

(iv) Sketch in a single diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

Q18

The function f is defined by $f : x \mapsto 5 - 3\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [3]

(iii) State, with a reason, whether f has an inverse. [1]

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Q19

The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x-b)^2 - c$. [3]

(ii) State the range of f. [1]

(iii) Find the set of values of x for which $f(x) < 21$. [3]

The function g is defined by $g : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

(iv) Find the value of the constant k for which the equation $gf(x) = 0$ has two equal roots. [4]

Q20

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 4x - 2x^2,$$
$$g : x \mapsto 5x + 3.$$

(i) Find the range of f. [2]

(ii) Find the value of the constant k for which the equation $gf(x) = k$ has equal roots. [3]

Q21

The function $f : x \mapsto 4 - 3\sin x$ is defined for the domain $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $f(x) = 2$. [3]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [2]

The function $g : x \mapsto 4 - 3\sin x$ is defined for the domain $\frac{1}{2}\pi \leqslant x \leqslant A$.

(iv) State the largest value of A for which g has an inverse. [1]

(v) For this value of A, find the value of $g^{-1}(3)$. [2]

Q22

The function $f : x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = f(x)$. [4]

(ii) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of f. [1]

The function $g : x \mapsto 2x^2 - 8x + 14$ is defined for $x \geqslant A$.

(iv) Find the smallest value of A for which g has an inverse. [1]

(v) For this value of A, find an expression for $g^{-1}(x)$ in terms of x. [3]

Q23

Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 3,$$
$$g : x \mapsto x^2 - 2x.$$

Express $gf(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [5]

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Q24

A function f is defined by $f : x \mapsto 3 - 2\tan(\frac{1}{2}x)$ for $0 \leqslant x < \pi$.

(i) State the range of f. [1]

(ii) State the exact value of $f(\frac{2}{3}\pi)$. [1]

(iii) Sketch the graph of $y = f(x)$. [2]

(iv) Obtain an expression, in terms of x, for $f^{-1}(x)$. [3]

Q25

The function f is defined by

$$f(x) = x^2 - 4x + 7 \quad \text{for } x > 2.$$

(i) Express $f(x)$ in the form $(x - a)^2 + b$ and hence state the range of f. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by

$$g(x) = x - 2 \quad \text{for } x > 2.$$

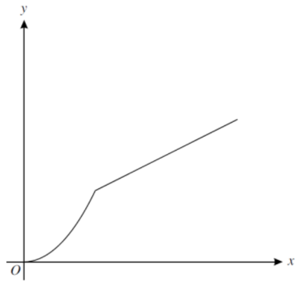
The function h is such that f = hg and the domain of h is $x > 0$.

(iii) Obtain an expression for $h(x)$. [1]

Q26

The diagram shows the function f defined for $0 \leqslant x \leqslant 6$ by

$$x \mapsto \tfrac{1}{2}x^2 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$x \mapsto \tfrac{1}{2}x + 1 \quad \text{for } 2 < x \leqslant 6.$$



(i) State the range of f.

(ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$.

(iii) Obtain expressions to define $f^{-1}(x)$, giving the set of values of x for which each expr valid.

Q27

Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 1,$$
$$g : x \mapsto x^2 - 2.$$

(i) Find and simplify expressions for $fg(x)$ and $gf(x)$. [2]

(ii) Hence find the value of a for which $fg(a) = gf(a)$. [3]

(iii) Find the value of b ($b \neq a$) for which $g(b) = b$. [2]

(iv) Find and simplify an expression for $f^{-1}g(x)$. [2]

The function h is defined by

$$h : x \mapsto x^2 - 2, \quad \text{for } x \leqslant 0.$$

(v) Find an expression for $h^{-1}(x)$. [2]

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Q28

The function f is defined by $f : x \mapsto \dfrac{x+3}{2x-1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{2}$.

(i) Show that $ff(x) = x$. [3]

(ii) Hence, or otherwise, obtain an expression for $f^{-1}(x)$. [2]

Q29

Functions f and g are defined by

$$f : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$
$$g : x \mapsto 2(x-1)^3 + 8, \quad x > 1.$$

(i) Evaluate $fg(2)$. [2]

(ii) Sketch in a single diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

(iii) Obtain an expression for $g'(x)$ and use your answer to explain why g has an inverse. [3]

(iv) Express each of $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x. [4]

Q30

Functions f and g are defined by

$$f : x \mapsto 2x^2 - 8x + 10 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$g : x \mapsto x \quad \text{for } 0 \leqslant x \leqslant 10.$$

(i) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) State the domain of f^{-1}. [1]

(iv) Sketch on the same diagram the graphs of $y = f(x)$, $y = g(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [4]

(v) Find an expression for $f^{-1}(x)$. [3]

Q31

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 3x + a,$$
$$g : x \mapsto b - 2x,$$

where a and b are constants. Given that $ff(2) = 10$ and $g^{-1}(2) = 3$, find

(i) the values of a and b, [4]

(ii) an expression for $fg(x)$. [2]

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Q32

The function $f : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geqslant p$, where k and p are constants.

(i) Express $f(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. [2]

(ii) State the range of f in terms of k. [1]

(iii) State the smallest value of p for which f is one-one. [1]

(iv) For the value of p found in part **(iii)**, find an expression for $f^{-1}(x)$ and state the domain of f^{-1}, giving your answers in terms of k. [4]

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Answers:

Q1: (i) a=2, b=-3 (ii) 4x-9

Q2: (i) a=16, b=-4 (ii) (4,16) (iii) $-2 \leq x \leq 10$ (iv) $x \leq 16$ $g^{-1} \geq -4$ (v) $g^{-1}(x) = \sqrt{(16-x)}$

Q3: (i) $x < -3$ & $x > 5$ (ii) $f(x) \geq -1$ (iv)

Q4:

10

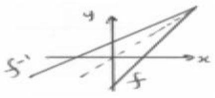
(i) fg(x) = g first, then f
= 8/(2-x) – 5 = 7
→ $x = 1\frac{1}{3}$

(or f(A)=7, A = 6, g(x) = 6, → $x = 1\frac{1}{3}$)

(ii) $f^{-1} = ½(x+5)$
Makes y the subject y = 4÷(2-x)
→ $g^{-1} = 2 – (4÷x)$

(iii) 2-4/x = ½ (x+5)
→ $x^2+x+8=0$
Use of b^2-4ac → Negative value
→ No roots.

(iv)



Q5

6 (i) $5s^2 + 3c^2 = 5s^2 + 3(1 – s^2)$
→ $3 + 2\sin^2 x$ a = 3, b = 2

(ii) $3 + 2s^2 = 7s$
Sets to 0 and solves.
s = ½ or s = 3
Only values are $\pi/6$ and $5\pi/6$

(iii) Minimum value = "a" = 3
Maximum value is "a + b" = 5

Range $3 \leq f(x) \leq 5$

Q6

$f : x \to 2x – a$

(i) ff(x) = 11, 2(2x – 3) – 3 = 11
[or backwards 2x – 3 = 11, x = 7,
2x – 3 = 7 (M1), (M1)]
→ x = 5

(ii) $2x – a = x^2 – 6x \to x^2 – 8x + a = 0$
Use of $b^2 – 4ac = 0$
→ a = 16 (or inspection)

(iii) $x^2 – 6x = (x – 3)^2 – 9$
→ p = 3, q = 9

(iv) $y = (x – 3)^2 – 9$
$x = \pm\sqrt{(y + 9)} + 3$
$y = h^{-1}(x) = \sqrt{(x + 9)} + 3$

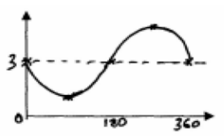
Domain of h^{-1} = {x: $x \geq –9$}

Q7:

$f : x \to 3 – 2\sin x$ for $0° \leq x \leq 360°$.

(i) Range $1 \leq f(x) \leq 5$

(ii)



g: $x \to 3 – 2\sin x$ for $0° \leq x \leq A°$

(iii) Maximum value of A = 90 or ½ π

(iv) $y = 3 – 2 \sin x$

$g^{-1}(x) = \sin^{-1}\left(\frac{3-x}{2}\right)$

Q8

8. $f : x \mapsto (2x – 3)^3 – 8$

(i) $f'(x) = 3(2x - 3)^2 \times 2$

Since ( )² is +ve, f'(x) +ve for all x
Therefore an increasing function.
(or t.p. at (1.5, -8) M1. Compares with y values at 2, or 4 + conclusion A1)

(ii) $y = (2x – 3)^3 – 8,$
$2x – 3 = \sqrt[3]{(y+8)}$
$\to f^{-1}(x) = \frac{\sqrt[3]{(x+8)}+3}{2}$

Domain $-7 \leq x \leq 117$

Q9

11 $f : x \mapsto k – x$
$g : x \mapsto \frac{9}{x+2}$

(i) $k – x = \frac{9}{x+2}$
→ $x^2 + (2 – k)x + 9 – 2k = 0$
Use of $b^2 – 4ac$
→ a = 4 or -8
k = 4, root is $\frac{-b}{2a} = 1$
k = –8, root is – 5.

(ii) $fg(x) = 6 – \frac{9}{x+2}$
Equates and solves with 5
x = 7
[or fg(x) = 5 → g(x) = 1 → x = 7]

(iii) $y = \frac{9}{x+2}$ → $x = \frac{9}{y} – 2$
$g^{-1}(x) = \frac{9}{x} – 2$ or $\frac{9-2x}{x}$

Q10:

(i) $x^2 – 3x – 4$ → –1 and 4
→ $x < –1$ and $x > 4$

(ii) $x^2 – 3x = (x – \frac{3}{2})^2 – \frac{9}{4}$

(iii) f(x) (or y) $\geq -\frac{9}{4}$

(iv) No inverse – not 1 : 1.

(v) Quadratic in $\sqrt{x}$.
Solution → $\sqrt{x}$ = 5 or –2
→ x = 25

Q11

(i) $f'(x) = –6(2x+3)^{-2}$ × 2
Always –ve → Decreasing

(ii) $y = \frac{6}{2x+3}$
→ $f^{-1}(x) = \frac{1}{2}\left(\frac{6}{x} – 3\right)$
Domain of f^{-1}: $0 < x \leq 2$

(iii)

(iv) $fg(x) = \frac{6}{x+3}$
= 1.5 → x = 1
[or , using f^{-1}, → g(x) = ½, ⇒ x = 1.]
[M1 M1 A1]

Q12

. $f(x) = 2x^2 – 8x + 11$

(i) $f(x) = 2(x – 2)^2 + 3$

(ii) Range is ≥ 3

(iii) Not 1:1(2 x-values for 1 y-value) or curve is quadratic – or has a minimum value

(iv) A = 2

(v) $y = 2(x – 2)^2 + 3$
→ $\frac{y-3}{2} = (x – 2)^2$
→ $x = 2 \pm \sqrt{\left(\frac{y-3}{2}\right)}$
→ $g^{-1}(x) = 2 – \sqrt{\left(\frac{x-3}{2}\right)}$

Range of g^{-1} ≤ 2

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Q13:

$x \mapsto (3x+2)^3 - 5$

(i) $f'(x) = 9(3x+2)^2$ or $81x^2 + 108x + 36$.
Because of $(\ )^2$ always +ve
Therefore an increasing function.

(ii) $y = (3x+2)^3 - 5$

$\sqrt[3]{y+5} = 3x+2$

$f^{-1}(x) = \dfrac{\sqrt[3]{x+5} - 2}{3}$

Domain of f^{-1} = range of f
$\rightarrow x \geqslant 3$

Q14

$f: x \mapsto 4x - 2k$, $g: x \mapsto \dfrac{9}{2-x}$

(i) $fg(x) = \dfrac{36}{2-x} - 2k = x$

$x^2 + 2kx - 2x + 36 - 4k$

$(2k-2)^2 = 4(36-4k)$

$k = 5$ or -7

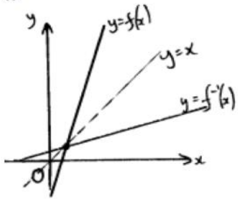
(ii) $x^2 + 8x + 16 = 0$, $x^2 - 16x + 64 = 0$

$x = -4$ or $x = 8$.

Q15

$f: x \mapsto 3x - 2$

(i)



(ii) $gf(x) = 6(3x-2) - (3x-2)^2$
$= -9x^2 + 30x - 16$
$d/dx = -18x + 30$
$= 0$ when $x = 5/3$
$\rightarrow$ Max of 9

$(gf(x) = 9 - (3x-5)^2 \rightarrow$ Max 9)

(iii) $6x - x^2 = 9 - (x-3)^2$

(iv) $y = 9 - (3-x)^2$
$3 - x = \pm\sqrt{9-y}$
$\rightarrow h^{-1}(x) = 3 + \sqrt{(9-x)}$

Q16:

(i) $2x^2 - 12x + 13 = 2(x-3)^2 - 5$

(ii) Symmetrical about $x = 3$. $A = 6$.

(iii) One limit is -5
Other limit is 13

(iv) Inverse since 1:1 $(4 > 3)$.

(v) Makes x the subject of the equation
Order of operations correct

$\rightarrow \sqrt{\dfrac{x+5}{2}} + 3$

Q17

$f: x \mapsto 2x + 1$, $x \in \mathbb{R}$, $x > 0$
$g: x \mapsto \dfrac{2x-1}{x+3}$, $x \in \mathbb{R}$, $x \neq -3$.

(i) $gf(x) = \dfrac{2(2x+1)-1}{2x+1+3} = \dfrac{4x+1}{2x+4}$
Equates to $x \rightarrow 2x^2 = 1$
$\rightarrow x = \tfrac{1}{2}\sqrt{2}$

(ii) $f^{-1}(x) = \tfrac{1}{2}(x-1)$
To find $g^{-1}(x)$, make x the subject
Order must be correct
$\rightarrow g^{-1}(x) = \dfrac{-1-3x}{x-2}$ or $\dfrac{1+3x}{2-x}$

(iii) $\dfrac{1+3x}{2-x} = x \rightarrow x^2 + x + 1 = 0$
Looks at $b^2 - 4ac$
$\rightarrow$ negative $\rightarrow$ no roots.

Q Q18

(i) $2 \leqslant f(x) \leqslant 8$

(ii) $x \mapsto 5 - 3\sin 2x$

(iii) No inverse – not 1 : 1.

Q19

(i) $2x^2 - 12x + 7 = 2(x-3)^2 - 11$

(ii) Range of f $\geqslant -11$

(iii) $2x^2 - 12x + 7 < 21$
$\rightarrow 2x^2 - 12x - 14$ or
$2(x-3)^2 < 32$
$\rightarrow$ end-values of 7 or -1
$\rightarrow -1 < x < 7$

(iv) $gf(x) = 2(2x^2 - 12x + 7) + k = 0$
Use of $b^2 - 4ac$
$\rightarrow 24^2 - 16(14 + k)$
$\rightarrow k = 22$

Q20:

$f: x \mapsto 4x - 2x^2$,
$g: x \mapsto 5x + 3$.

(i) Turning point at $x = 1$.
Range is $\leqslant 2$.

(ii) $gf(x) = 5(4x - 2x^2) + 3$
$= k$ and use of $b^2 - 4ac$
$\rightarrow k = 13$

Q21

$f: x \mapsto 4 - 3\sin x$

(i) $4 - 3\sin x = 2 \rightarrow \sin x = \tfrac{2}{3}$
$\rightarrow x = 0.730$ or 2.41

(ii)

(iii) $k < 1$, $k > 7$.

(iv) $A = \dfrac{3\pi}{2}$.

(v) $\sin x = \tfrac{1}{3}$ – or using inverse
$g^{-1}(3) = 2.80$

Q22

$f: x \mapsto 2x^2 - 8x + 14$

(i) $y + kx = 12$, Sim Eqns.
$\rightarrow 2x^2 - 8x + kx + 2 = 0$
Use of $b^2 - 4ac$
$\rightarrow (k-8)^2 = 16 \rightarrow k = 12$ or 4.

(ii) $2x^2 - 8x + 14 = 2(x-2)^2 + 6$

(iii) Range of f $\geqslant 6$.

(iv) Smallest $A = 2$

(v) Makes x the subject
Order of operations correct.

$g^{-1}(x) = \sqrt{\dfrac{x-6}{2}} + 2$

Q23

$f: x \mapsto 2x + 3$,
$g: x \mapsto x^2 - 2x$,
$gf(x) = (2x+3)^2 - 2(2x+3)$
$= 4x^2 + 8x + 3$
$= 4(x+1)^2 - 1$

Q24:

$x \mapsto 3 - 2\tan(\tfrac{1}{2}x)$

(i) Range of f ≤ 3

(ii) $f(\tfrac{2}{3}\pi) = 3 - 2\sqrt{3}$

(iii)

(iv) $y = 3 - 2\tan\left(\dfrac{x}{2}\right)$

$\rightarrow f^{-1}(x) = 2\tan^{-1}\left(\dfrac{3-x}{2}\right)$

Q25

(i) $(x-2)^2$
$(x-2)^2 + 3$
$f(x) > 3$

(ii) $x - 2 = (\pm)\sqrt{y-3}$

$f^{-1}(x) = 2 + \sqrt{x-3}$
domain is $x > 3$

(iii) $h(x) = x^2 + 3$

Q26

(i) Range is $0 < f(x) < 4$, 0 to 4

(ii) $y = x$ drawn or implied
Correct sketch of f^{-1}

(iii) $(x \mapsto)\sqrt{2x}$ for $0 < x < 2$
$(x \mapsto) 2x - 2$ for $2 < x < 4$

Q27

(i) $fg(x) = 2x^2 - 3$, $gf(x) = 4x^2 + 4x - 1$

(ii) $2a^2 - 3 = 4a^2 + 4a - 1 \Rightarrow 2a^2 + 4a + 2 = 0$
$(a+1)^2 = 0$
$a = -1$

(iii) $b^2 - b - 2 = 0 \rightarrow (b+1)(b-2) = 0$
$b = 2$ Allow $b = -1$ in addition

(iv) $f^{-1}(x) = \dfrac{1}{2}(x-1)$

$f^{-1}g(x) = \dfrac{1}{2}(x^2 - 3)$

(v) $x = (\pm)\sqrt{y+2}$
$h^{-1}(x) = -\sqrt{x+2}$

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Q28:

(i) $f(x)=\dfrac{x+3}{2x-1}$

$ff(x)=\dfrac{\dfrac{x+3}{2x-1}+3}{\dfrac{2(x+3)}{2x-1}-1}=\dfrac{7x}{7}=x$

(ii) $y=\dfrac{x+3}{2x-1}$

$\rightarrow\ 2xy-y=x+3$

$\rightarrow\ x(2y-1)=y+3$

$\rightarrow\ f^{-1}(x)=\dfrac{x+3}{2x-1}$

or since $ff(x)=x$,

$f^{-1}(x)=f(x)=\dfrac{x+3}{2x-1}$ (M1, A1)

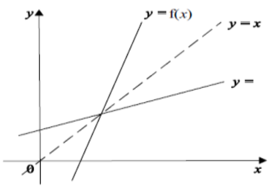
Q29

$f:x\mapsto 3x-4$ $\quad g:x\mapsto 2(x-1)^3+8$

(i) $fg(2)=f(10)=26$

$f^{-1}(x)$

(ii)



(iii) $g'(x)=6(x-1)^2$

$g'(x)>\rightarrow$ no turning points

$\rightarrow$ g is 1 : 1, g has an inverse.

(iv) $f^{-1}(x)=\dfrac{x+4}{3}$

Attempt at making x

Order correct. -8, $\div 2$, $\sqrt[3]{\ }$, $+1$

$g^{-1}(x)=\sqrt[3]{\dfrac{x-8}{2}}+1$

Q30:

(i) $2(x-2)^2+2$

(ii) $2\le f(x)\le 10$ oe

(iii) $2\le x\le 10$

(iv) $f(x)$: ≈ half parabola from $(0,10)$ to $(2,2)$

$g(x)$: line through 0 at ≈ 45°

$f^{-1}(x)$: reflection of *their* $f(x)$ in $g(x)$

Everything totally correct

(v) $(x-2)^2=\dfrac{1}{2}(y-2)$

$x=2\pm\sqrt{\dfrac{1}{2}(y-2)}$

$f^{-1}(x)=2-\sqrt{\dfrac{1}{2}(x-2)}$

Q31

$f:\ x\mapsto 3x+a,\ \ g:\ x\mapsto b-2x$

(i) $f^2(x)=3(3x+a)+a$

$f^2(2)\ =\ 18+4a=10\ \ \rightarrow\ a=-2$

$g^{-1}(x)=\dfrac{b-x}{2}\ \rightarrow\ \dfrac{b-2}{2}=3\qquad b=8$

or $\ g(3)=2\ \rightarrow\ b-6=2\qquad b=8$

(ii) $fg(x)=3(b-2x)+a$

$=\ 22-6x$

Q32

(i) $(x-2)^2-4+k$

(ii) $f(x)>k-4$ or $[k-4,\infty]$ or $(k-4,\infty)$ oe

(iii) smallest value of $p=2$

(iv) $x-2=(\pm)\sqrt{y+4-k}$

$x=2+\sqrt{y+4-k}$

$f^{-1}(x)=2+\sqrt{x+4-k}$

Domain is $x>k-4$ or $[k-4,\infty]$

or $(k-4,\infty)$ oe