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Coordinate Geometry P1

Q1

(i) Sketch the graph of the curve $y = 3\sin x$, for $-\pi \leqslant x \leqslant \pi$. [2]

The straight line $y = kx$, where k is a constant, passes through the maximum point of this curve for $-\pi \leqslant x \leqslant \pi$.

(ii) Find the value of k in terms of π. [2]

(iii) State the coordinates of the other point, apart from the origin, where the line and the curve intersect. [1]

Q2

The line L_1 has equation $2x + y = 8$. The line L_2 passes through the point $A\ (7, 4)$ and is perpendicular to L_1.

(i) Find the equation of L_2. [4]

(ii) Given that the lines L_1 and L_2 intersect at the point B, find the length of AB. [4]

Q3

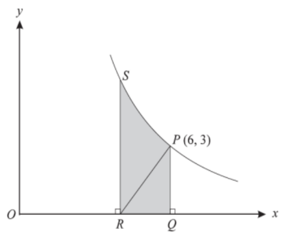
The curve $y = 9 - \dfrac{6}{x}$ and the line $y + x = 8$ intersect at two points. Find

(i) the coordinates of the two points, [4]

(ii) the equation of the perpendicular bisector of the line joining the two points. [4]

Q4

The diagram shows part of the graph of $y = \dfrac{18}{x}$ and the normal to the curve at $P\ (6, 3)$. This normal meets the x-axis at R. The point Q on the x-axis and the point S on the curve are such that PQ and SR are parallel to the y-axis.

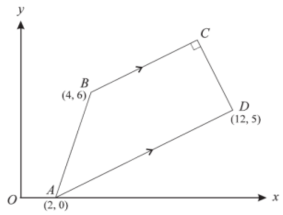


(i) Find the equation of the normal at P and show that R is the point $(4\frac{1}{2}, 0)$. [5]

Q5

Find the coordinates of the points of intersection of the line $y + 2x = 11$ and the curve $xy = 12$. [4]

Q6



The diagram shows a trapezium $ABCD$ in which BC is parallel to AD and angle $BCD = 90°$. The coordinates of A, B and D are $(2, 0)$, $(4, 6)$ and $(12, 5)$ respectively.

(i) Find the equations of BC and CD. [5]

(ii) Calculate the coordinates of C. [2]

Q7

The equation of a curve is $y = x^2 - 4x + 7$ and the equation of a line is $y + 3x = 9$. The curve and the line intersect at the points A and B.

(i) The mid-point of AB is M. Show that the coordinates of M are $(\frac{1}{2}, 7\frac{1}{2})$. [4]

(ii) Find the coordinates of the point Q on the curve at which the tangent is parallel to the line $y + 3x = 9$. [3]

(iii) Find the distance MQ. [1]

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Q8

A curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{(4x-3)}}$ and $P(3, 3)$ is a point on the curve.

(i) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by = c$. [3]

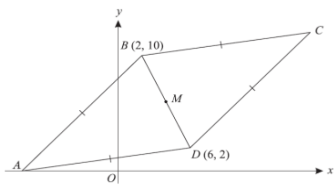
(ii) Find the equation of the curve. [4]

Q9

A curve is such that $\frac{dy}{dx} = 2x^2 - 5$. Given that the point $(3, 8)$ lies on the curve, find the equation of the curve. [4]

Q10

The diagram shows a rhombus $ABCD$. The points B and D have coordinates $(2, 10)$ and $(6, 2)$ respectively, and A lies on the x-axis. The mid-point of BD is M. Find, by calculation, the coordinates of each of M, A and C. [6]



Q11

A curve has equation $y = \frac{4}{\sqrt{x}}$.

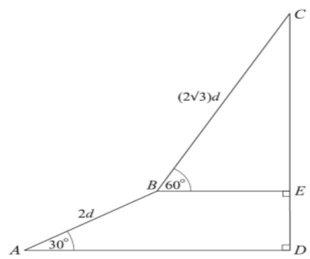
(i) The normal to the curve at the point $(4, 2)$ meets the x-axis at P and the y-axis at Q. Find the length of PQ, correct to 3 significant figures. [6]

Q12

In the diagram, $ABED$ is a trapezium with right angles at E and D, and CED is a straight line. The lengths of AB and BC are $2d$ and $(2\sqrt{3})d$ respectively, and angles BAD and CBE are 30° and 60° respectively.

(i) Find the length of CD in terms of d. [2]

(ii) Show that angle $CAD = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$. [3]



Q13

Three points have coordinates $A(2, 6)$, $B(8, 10)$ and $C(6, 0)$. The perpendicular bisector of AB meets the line BC at D. Find

(i) the equation of the perpendicular bisector of AB in the form $ax + by = c$, [4]

(ii) the coordinates of D. [4]

Q14

The equation of a curve is $xy = 12$ and the equation of a line l is $2x + y = k$, where k is a constant.

(i) In the case where $k = 11$, find the coordinates of the points of intersection of l and the curve. [3]

(ii) Find the set of values of k for which l does not intersect the curve. [4]

(iii) In the case where $k = 10$, one of the points of intersection is $P(2, 6)$. Find the angle, in degrees correct to 1 decimal place, between l and the tangent to the curve at P. [4]

Q15

A curve has equation $y = \frac{k}{x}$. Given that the gradient of the curve is -3 when $x = 2$, find the value of the constant k. [3]

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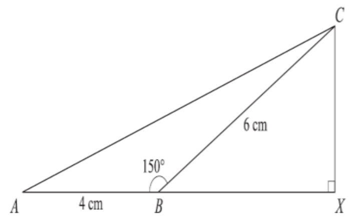
Q16

The curve $y^2 = 12x$ intersects the line $3y = 4x + 6$ at two points. Find the distance between the two points. [6]

Q17

In the diagram, ABC is a triangle in which $AB = 4$ cm, $BC = 6$ cm and angle $ABC = 150°$. The line CX is perpendicular to the line ABX.



(i) Find the exact length of BX and show that angle $CAB = \tan^{-1}\left(\frac{3}{4+3\sqrt{3}}\right)$. [4]

(ii) Show that the exact length of AC is $\sqrt{(52 + 24\sqrt{3})}$ cm. [2]

Q18

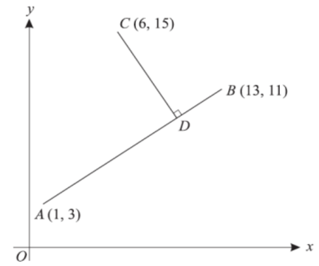
A curve is such that $\frac{dy}{dx} = \frac{4}{\sqrt{(6-2x)}}$, and $P(1, 8)$ is a point on the curve.

(i) The normal to the curve at the point P meets the coordinate axes at Q and at R. Find the coordinates of the mid-point of QR. [5]

(ii) Find the equation of the curve. [4]

Q19

The three points $A(1, 3)$, $B(13, 11)$ and $C(6, 15)$ are shown in the diagram. The perpendicular from C to AB meets AB at the point D. Find

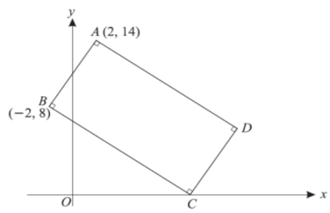


(i) the equation of CD, [3]

(ii) the coordinates of D. [4]

Q20

The diagram shows a rectangle $ABCD$. The point A is $(2, 14)$, B is $(-2, 8)$ and C lies on the x-axis. Find

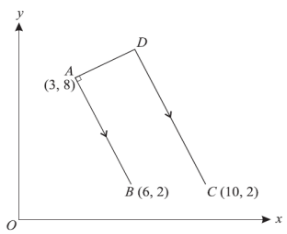


(i) the equation of BC, [4]

(ii) the coordinates of C and D. [3]

Q21

The three points $A(3, 8)$, $B(6, 2)$ and $C(10, 2)$ are shown in the diagram. The point D is such that the line DA is perpendicular to AB and DC is parallel to AB. Calculate the coordinates of D. [7]



Q22

A curve is such that $\frac{dy}{dx} = 4 - x$ and the point $P(2, 9)$ lies on the curve. The normal to the curve at P meets the curve again at Q. Find

(i) the equation of the curve, [3]

(ii) the equation of the normal to the curve at P, [3]

(iii) the coordinates of Q. [3]

Q23

In the triangle ABC, $AB = 12$ cm, angle $BAC = 60°$ and angle $ACB = 45°$. Find the exact length of BC. [3]

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Q24

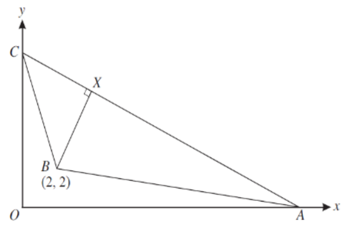
The equation of a curve C is $y = 2x^2 - 8x + 9$ and the equation of a line L is $x + y = 3$.

(i) Find the x-coordinates of the points of intersection of L and C. [4]

(ii) Show that one of these points is also the stationary point of C. [3]

Q25

In the diagram, the points A and C lie on the x- and y-axes respectively and the equation of AC is $2y + x = 16$. The point B has coordinates $(2, 2)$. The perpendicular from B to AC meets AC at the point X.



(i) Find the coordinates of X. [4]

The point D is such that the quadrilateral $ABCD$ has AC as a line of symmetry.

(ii) Find the coordinates of D. [2]

(iii) Find, correct to 1 decimal place, the perimeter of $ABCD$. [3]

The equation of a curve is $y = 5 - \frac{\ }{x}$.

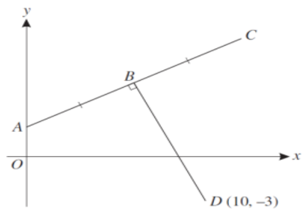
(i) Show that the equation of the normal to the curve at the point $P(2, 1)$ is $2y + x = 4$. [4]

This normal meets the curve again at the point Q.

(ii) Find the coordinates of Q. [3]

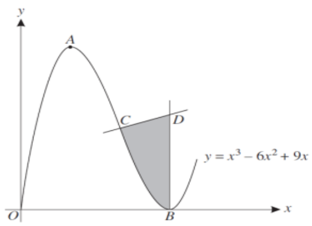
(iii) Find the length of PQ. [2]

Q27



The diagram shows points A, B and C lying on the line $2y = x + 4$. The point A lies on the y-axis and $AB = BC$. The line from $D(10, -3)$ to B is perpendicular to AC. Calculate the coordinates of B and C. [7]

Q28



The diagram shows the curve $y = x^3 - 6x^2 + 9x$ for $x \geqslant 0$. The curve has a maximum point at A and a minimum point on the x-axis at B. The normal to the curve at $C(2, 2)$ meets the normal to the curve at B at the point D.

(i) Find the coordinates of A and B. [3]

(ii) Find the equation of the normal to the curve at C. [3]

Q29

A curve is such that $\frac{dy}{dx} = k - 2x$, where k is a constant.

(i) Given that the tangents to the curve at the points where $x = 2$ and $x = 3$ are perpendicular, find the value of k. [4]

(ii) Given also that the curve passes through the point $(4, 9)$, find the equation of the curve. [3]

Q30

The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$. Given that the curve passes through the point $(4, 6)$, find the equation of the curve. [4]

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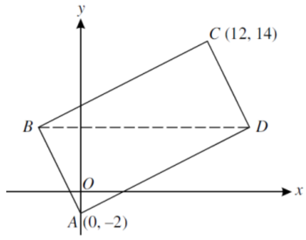
Q31

The diagram shows a rectangle $ABCD$. The point A is $(0, -2)$ and C is $(12, 14)$. The diagonal BD is parallel to the x-axis.



(i) Explain why the y-coordinate of D is 6. [1]

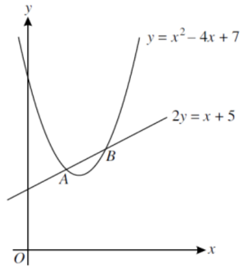
The x-coordinate of D is h.

(ii) Express the gradients of AD and CD in terms of h. [3]

(iii) Calculate the x-coordinates of D and B. [4]

(iv) Calculate the area of the rectangle $ABCD$. [3]

Q32



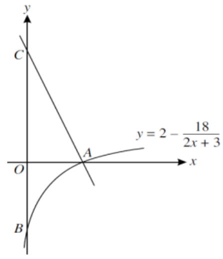
(i) The diagram shows the line $2y = x + 5$ and the curve $y = x^2 - 4x + 7$, which intersect at the points A and B. Find

(a) the x-coordinates of A and B, [3]

(b) the equation of the tangent to the curve at B, [3]

(c) the acute angle, in degrees correct to 1 decimal place, between this tangent and the line $2y = x + 5$. [3]

Q33

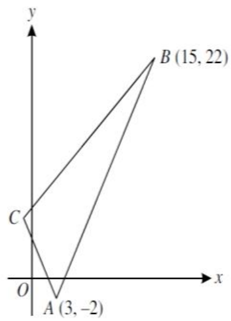


The diagram shows part of the curve $y = 2 - \dfrac{18}{2x+3}$, which crosses the x-axis at A and the y-axis at B. The normal to the curve at A crosses the y-axis at C.

(i) Show that the equation of the line AC is $9x + 4y = 27$. [6]

(ii) Find the length of BC. [2]

Q34



The diagram shows a triangle ABC in which A is $(3, -2)$ and B is $(15, 22)$. The gradients of AB, AC and BC are $2m$, $-2m$ and m respectively, where m is a positive constant.

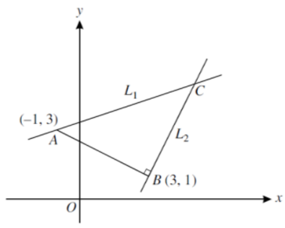
(i) Find the gradient of AB and deduce the value of m. [2]

(ii) Find the coordinates of C. [4]

The perpendicular bisector of AB meets BC at D.

(iii) Find the coordinates of D. [4]

Q35



In the diagram, A is the point $(-1, 3)$ and B is the point $(3, 1)$. The line L_1 passes through A and is parallel to OB. The line L_2 passes through B and is perpendicular to AB. The lines L_1 and L_2 meet at C. Find the coordinates of C. [6]

Q36

The equation of a curve is such that $\dfrac{dy}{dx} = \dfrac{6}{\sqrt{(3x-2)}}$. Given that the curve passes through the point $P\,(2, 11)$, find

(i) the equation of the normal to the curve at P, [3]

(ii) the equation of the curve. [4]

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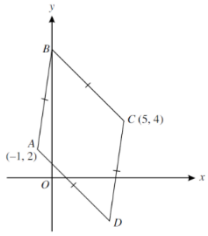
Q37



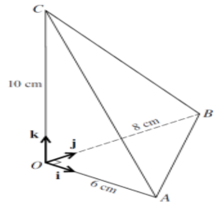
The diagram shows a rhombus $ABCD$ in which the point A is $(-1, 2)$, the point C is $(5, 4)$ and the point B lies on the y-axis. Find

(i) the equation of the perpendicular bisector of AC, [3]

(ii) the coordinates of B and D, [3]

(iii) the area of the rhombus. [3]

Q38



The diagram shows a pyramid $OABC$ with a horizontal base OAB where $OA = 6$ cm, $OB = 8$ cm and angle $AOB = 90°$. The point C is vertically above O and $OC = 10$ cm. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OB and OC as shown.

Use a scalar product to find angle ACB. [6]

Q39

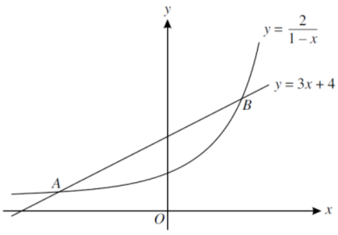
The equation of a curve is $y = 3 + 4x - x^2$.

(i) Show that the equation of the normal to the curve at the point $(3, 6)$ is $2y = x + 9$. [4]

(ii) Given that the normal meets the coordinate axes at points A and B, find the coordinates of the mid-point of AB. [2]

(iii) Find the coordinates of the point at which the normal meets the curve again. [4]

Q40



The diagram shows part of the curve $y = \dfrac{2}{1-x}$ and the line $y = 3x + 4$. The curve and the line meet at points A and B.

(i) Find the coordinates of A and B. [4]

(ii) Find the length of the line AB and the coordinates of the mid-point of AB. [3]

Q41

Points A, B and C have coordinates $(2, 5)$, $(5, -1)$ and $(8, 6)$ respectively.

(i) Find the coordinates of the mid-point of AB. [1]

(ii) Find the equation of the line through C perpendicular to AB. Give your answer in the form $ax + by + c = 0$. [3]

Q42

(i) Express $2x^2 - 4x + 1$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the minimum point, A, on the curve $y = 2x^2 - 4x + 1$. [4]

The line $x - y + 4 = 0$ intersects the curve $y = 2x^2 - 4x + 1$ at points P and Q. It is given that the coordinates of P are $(3, 7)$.

(ii) Find the coordinates of Q. [3]

(iii) Find the equation of the line joining Q to the mid-point of AP. [3]

Q43

The line L_1 passes through the points $A\,(2, 5)$ and $B\,(10, 9)$. The line L_2 is parallel to L_1 and passes through the origin. The point C lies on L_2 such that AC is perpendicular to L_2. Find

(i) the coordinates of C, [5]

(ii) the distance AC. [2]

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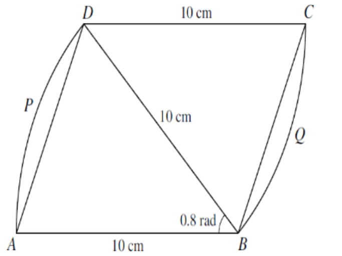
Q44

The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, meets the x-axis at P and the y-axis at Q. Given that $PQ = \sqrt{(45)}$ and that the gradient of the line PQ is $-\frac{1}{2}$, find the values of a and b. [5]

Q45



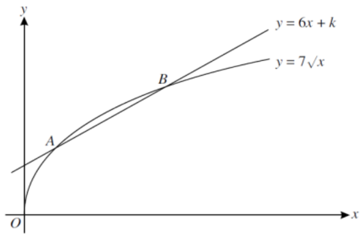
In the diagram, $ABCD$ is a parallelogram with $AB = BD = DC = 10$ cm and angle $ABD = 0.8$ radians. APD and BQC are arcs of circles with centres B and D respectively.

(i) Find the area of the parallelogram $ABCD$. [2]

(ii) Find the area of the complete figure $ABQCDP$. [2]

(iii) Find the perimeter of the complete figure $ABQCDP$. [2]

Q46



The diagram shows the curve $y = 7\sqrt{x}$ and the line $y = 6x + k$, where k is a constant. The curve and the line intersect at the points A and B.

(i) For the case where $k = 2$, find the x-coordinates of A and B. [4]

(ii) Find the value of k for which $y = 6x + k$ is a tangent to the curve $y = 7\sqrt{x}$. [2]

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Answers:

Q1: (i) x=π/2, y=3, k=6/π (ii) (-π/2,-3)

Q2: (i) y-4=0.5(x-7) (ii) 4.47

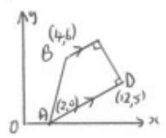
Q3: (i) (2,6) & (-3,11) (ii) $y = x + 9$ or $y - 8\frac{1}{2} = \left(x + \frac{1}{2}\right)$

Q4: $y = 2x - 9$

Q5: $\left(1\frac{1}{2}, 8\right)$ & $(4,3)$

Q6

5



(i) m of BC = ½
Eqn BC y-6=½(x-4)
m of CD = -2
eqn CD y-5=-2(x-12)

(ii) Sim eqns 2y=x+8 and y+2x=29
→ C (10,9)

Q7:

5 (i) $x^2 - 4x + 7 = 9 - 3x \rightarrow x^2 - x - 2 = 0$
Solution of this $x = 2$ or -1
→ (2, 3) and (–1, 12)
Mid point is M (½, 7½)

(ii) dy/dx = 2x – 4
Equate to m of line (–3) + solution
→ (½, 5¼)

(iii) Distance = 2¼

Q8

7 dy/dx = 6/√(4x – 3) P(3, 3)

(i) x = 3, m = 2. Perpendicular m = –½
→ y – 3 = –½ (x – 3) → x + 2y = 9

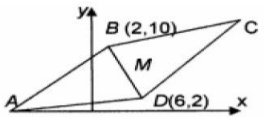
(ii) ∫ → $6(4x-3)^{½}$ ÷ ½ ÷ 4
y = 3(4x – 3) + *c*
Uses (3, 3) → *c* = –6

Q9

$y = \frac{2x^3}{3} - 5x$ (+ c)

(3,8) fits $y = \frac{2x^3}{3} - 5x + 5$

Q10:



M (4, 6)
m of *BD* = -2
M of *AC* = ½
Eqn of *AC* $y - 6 = \frac{1}{2}(x - 4)$

→ *x* = − 8 when *y* = 0 A(−8, 0)

→ *C* = (16, 12) by vector move etc.

Q11

$y = \frac{4}{\sqrt{x}}$

(i) dy/dx = $-2x^{-1.5}$
= –¼
m of normal = 4
Eqn of normal *y* – 2 = 4(*x* – 4)
P (3.5, 0) and Q (0, −14)

Length of *PQ* = $\sqrt{(3.5^2 + 14^2)}$
= 14.4

Q12

(i) 2*d*sin30 + 2*d*√3sin60
= 2*d*.½ + 2*d*√3.√3/2 = 4*d*

(ii) $\tan\theta = \frac{\text{ans to (i)}}{2d\cos 30 + 2\sqrt{3}d\cos 60}$

$\tan\theta = \frac{2}{\sqrt{3}}$

Q13

(i) *M* (5,8)
gradient of *AB* = $\frac{2}{3}$, Perp= $-\frac{3}{2}$
Equation $y - 8 = -\frac{3}{2}(x - 5)$
→ 2y + 3x = 31
(or locus method M1A1M1A1)

(ii) *BC*. y = 5(x-6) y = 5x – 30
Sim Eqns → (7,5)

Q14:

(i) 2x² + 12 = 11x or y²-11y+24=0

Solution → (1½, 8) and (4, 3)

Guesswork B1 for one, B3 for both.

(ii) 2x² - *kx* +12 =0
Use of *b*² - 4*ac*
*k*² < 96
-√96 < *k* < √96 or |*k* |<√96

(iii) gradient of 2*x* + *y* = *k* = −2
dy/d*x* = −12 / *x*² (= − 3)
Use of tangent for an angle
Difference = 8.1° or 8.2°

Q15

$\frac{dy}{dx} = -kx^{-2}$
Puts *x* = 2, *m* = –3
→ *k* = 12

Q16

5. $y^2 = 12x$ and $3y = 4x + 6$
Complete elimination of 1 variable.
→ $y^2 - 9y + 18 = 0$ or $4x^2 - 15x + 9 = 0$
solution → (¾, 3) and (3, 6)

Distance= $\sqrt{(3^2 + 2.25^2)}$ = 3.75

Q17

(i) *BX* = 6cos30 = 3√3
CX = 6sin30 = 3
Tan *CAB* = opp/adj = $\frac{3}{4+3\sqrt{3}}$
CAB = $\tan^{-1}\left(\frac{3}{4+3\sqrt{3}}\right)$

(ii) Pythagoras with his *AX* and *CX* or cosine rule used correctly

→ *AC* = $\sqrt{52 + 24\sqrt{3}}$

Q18:

(i) $\frac{dy}{dx} = \frac{4}{\sqrt{6-2x}}$
If *x* = 1, *m*=2 and perp *m* = −½.
→ $y - 8 = -\frac{1}{2}(x - 1)$ $(2y + x = 17)$
→ (0, 8½) and (17, 0)
→ M(8½, 4¼)

(ii) $y = \frac{4(6-2x)^{\frac{1}{2}}}{\frac{1}{2} \times -2} + c$
→ subs (1,8) → c = 16

Q19

(i) m of AB = 8/12
m of perpendicular = -12/8
eqn of *CD* $y - 15 = -\frac{3}{2}(x - 6)$

(ii) eqn of *AB* $y - 3 = \frac{2}{3}(x - 1)$
Sim eqns 2*y*+3*x*=48 and 3*y*=2*x*+7
→ *D* (10, 9)

Q20

m of *AB* = 1.5 (or 1½)
m of *BC* = −1 ÷ (m of *AB*) = −⅔
→ Eqn $y - 8 = -\frac{2}{3}(x + 2)$ or 3*y*+2*x*=20

Put *y* = 0 → *C* (10, 0)
Vector move → *D* (14, 6)
(or sim eqns 3*y*+2*x*=46 and 2*y*=3*x*−30)

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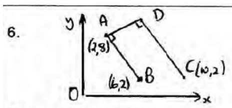
Q21:

6.



Gradient of $AB = -2$
Eqn of CD $y-2=-2(x-10)$
$(y+2x=22)$

Uses $m_1m_2=-1$
Eqn of DA $y-8=\frac{1}{2}(x-3)$
$(2y=x+13)$

Sim eqns → (6.2, 9.6)

Q22

3. (i) $y=4x-\frac{1}{2}x^2 + c$
Uses (2,9) → c = 3

(ii) grad of tan = 2, normal = −½
Eqn $y-9=-½(x-2)$

(iii) $y=4x-\frac{1}{2}x^2+3$, $2y+x=20$
eliminates y → $x^2-9x+14=0$
eliminates x → $2y^2-31y+117=0$

Soln of quadratic → $x=7$, $y=6.5$

Q23

Use of sine rule $\frac{12}{\sin 45}=\frac{x}{\sin 60}$

$\sin 60=\frac{\sqrt{3}}{2}$ and $\sin 45=\frac{1}{\sqrt{2}}$

$\to BC=6\sqrt{3}\sqrt{2}$ or $6\sqrt{6}$ or $\sqrt{216}$

Q24

(i) Eliminates y to get
$2x^2-7x+6=0$ or $2y^2-5y+3=0$
$\to (2x-3)(x-2)=0$
$\to x=2$ or $1½$

(ii) $dy/dx = 4x-8$
$=0$
$x=2$
or completes the square and states stationary at $x=2$.

Q25:

(i) Gradient of $AC = -½$
Perpendicular gradient = 2
Eqn of BX is $y-2=2(x-2)$
Sim Eqns $2y+x=16$ with $y=2x-2$
→ (4, 6)

(ii) X is mid-point of BD, D is (6, 10)

(iii) $AB=\sqrt{(14^2+2^2)}=\sqrt{200}$

$BC=\sqrt{(2^2+6^2)}=\sqrt{40}$

→ Perimeter $=2\sqrt{200}+2\sqrt{40}$

→ Perimeter = 40.9

Q26

$y=5-\frac{8}{x}$, $P(2,1)$

(i) $\frac{dy}{dx}=\frac{8}{x^2}$
m of tan = 2 m of normal = −½
Eqn of normal $y-1=-\frac{1}{2}(x-2)$
$\to 2y+x=4$

(ii) Sim eqns $2y+x=4$, $y=5-\frac{8}{x}$
$\to x^2+6x-16=0$ or $y^2-7y+6=0$
→ (−8, 6)

(iii) Length $=\sqrt{10^2+5^2}=\sqrt{125}$
→ 11.2 (accept $\sqrt{125}$ or $5\sqrt{5}$ etc)

Q27

m of AC = ½
Perpendicular gradient = −2
Eqn BD $y+3=-2(x-10)$
(or $y+2x=17$)

Sim. eqns BD with given eqn.
→ B (6,5)

Vector move (step) → C (12, 8)

Q28

(i) $\frac{dy}{dx}=3x^2-12x+9$

Solves $\frac{dy}{dx}=0$

→ A (1, 4), B (3, 0).

(ii) If $x=2$, $m=-3$
Normal has $m=\frac{1}{3}$

Eqn $y-2=\frac{1}{3}(x-2)$ or $3y=x+4$.

Q29:

$\frac{dy}{dx}=k-2x$

(i) At $x=2$, $m=(k-4)$ $x=3$
$m=(k-6)$
$(k-4)(k-6)=-1$
→ $k=5$

(ii) $y=kx-x^2\ (+c)$
Substitutes (4, 9)
→ $c=5$

Q30

$\frac{dy}{dx}=\frac{3}{\sqrt{x}}-x$

$(y)=6\sqrt{x}-\frac{x^2}{2}(+c)$

(4, 6) fits $6=12-8+c$
$\to c=2$

Q31

(i) y-coordinate same as the y-coordinate of the mid-point of AC.

(ii) m of $AD=\frac{8}{h}$ or $\frac{h-12}{8}$

m of $CD=\frac{8}{12-h}$ or $\frac{-h}{8}$

nb $AC=20$, $M(6, 6)$ $MD=10$ → $D(16, 6)$ and $B(-4, 6)$

(iii) Product of gradients = −1
$\to h^2-12h-64=0$
$\to h=16$ or -4
so $x_D=16$ and $x_B=-4$
or Pyth $h^2+8^2+8^2+(12-h)^2=400$

(iv) Area = √320 × √80
→ 160

(or Area = 2 × area of a triangle with base = BD, → 2 × ½ × 20 × 8 = 160)
(or matrix method)

Q32:

(i) (a) $2y=x+5$, $y=x^2-4x+7$
Sim equations → $2x^2-9x+9=0$
→ $x=3$ or $x=1½$.

(b) $\frac{dy}{dx}=2x-4$
$\to y-4=2(x-3)$

nb use of $y+4$ or x, y interchanged
M1 A0

(c) $m=2$ → angle of 63.4°
$m=½$ → angle of 26.6°
→ angle between = 37°

(**i**+2**j**).(2**i**+**j**) → 4=√5√5cosθ M1M1A1
or use of tan(A–B) M2A1
or Cosine rule with 3 sides found.

Q33

$y=2-\frac{18}{2x+3}$

(i) A is (3, 0)
$\frac{dy}{dx}=18(2x+3)^{-2}\times 2$
If $x=3$, $m=\frac{4}{9}$.
m of normal $=-\frac{9}{4}$
Equation of normal $y=-\frac{9}{4}(x-3)$
→ $4y+9x=27$

(ii) Normal meets y-axis at (0, 6¾)
Curve meets y-axis at (0, −4)
→ BC = 10¾

Q34

(i) y-step ÷ x-step = 2
→ $m=1$

(ii) Eqn of AC $y+2=-2(x-3)$
Eqn of BC $y-22=(x-15)$
Sim eqns $y+2x=4$, $y=x+7$
→ C(−1, 6)

(iii) M is (9, 10)
Perp gradient is −½
→ $2y+x=29$, $y=x+7$
Sim eqns → D (5, 12)

Q35

Gradient of L_1 is $\frac{1}{3}$.

Equation of L_1 is $y-3=\frac{1}{3}(x+1)$

Gradient of AB is $-\frac{1}{2}$. Perp = 2.

Equation of L_2 is $y-1=2(x-3)$.

Sim eqns $3y=x+10$, $y=2x-5$.

→ (5, 5)

Q36:

$\frac{dy}{dx}=\frac{6}{\sqrt{3x-2}}$

(i) $x=2$, tangent has gradient 3

→ normal has gradient $-\frac{1}{3}$

$\to y-11=-\frac{1}{3}(x-2)$

(ii) Integrate $\to 6\frac{\sqrt{3x-2}}{\frac{1}{2}}\div 3$

$\to y=4\sqrt{3x-2}+c$ through (2,11)

$\to y=4\sqrt{3x-2}+3$

Q37

(i) Mid-point of AC = (2, 3)
Gradient of AC = ⅓
Gradient of BD = −3
Equation $y-3=-3(x-2)$

(ii) If $x=0$, $y=9$, B (0, 9)
Vector move D (4, −3)

(iii) $AC=\sqrt{40}$
$BD=\sqrt{160}$
Area = 40
(or by matrix method M2 A1)

Q38

$\overrightarrow{AC}=-6\mathbf{i}+10\mathbf{k}$

$\overrightarrow{BC}=-8\mathbf{j}+10\mathbf{k}$

$\overrightarrow{AC}.\overrightarrow{BC}=100$

$\overrightarrow{AC}.\overrightarrow{BC}=\sqrt{136}\sqrt{164}\cos ACB$

Angle ACB = 48.0°

Q39

$y=4x-x^2+3$

(i) $\frac{dy}{dx}=4-2x$
At $x=3$, $m=-2$
Gradient of normal $=\frac{1}{2}$
Eqn of normal $y-6=\frac{1}{2}(x-3)$
$\to 2y=x+9$

(ii) Meets axes at $(0, \frac{9}{2})$ and (−9, 0)

Mid-point is $\left(\frac{-9}{2},\frac{9}{4}\right)$

(iii) $2y=x+9$, $y=4x-x^2+3$
$\to 2x^2-7x+3=0$ oe
→ (½, 4¾)

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Q40:

(i) $3x^2 + x - 2 = 0$
$(x+1)(3x-2) \rightarrow x = -1$ or ⅔
$(-1, 1)$, (⅔, 6)

(ii) $AB^2 = (5/3)^2 + 5^2$
$AB = 5.27(0\ldots)$
mid-point = $(-1/6, 7/2)$

Q41

(i) (3½, 2)

(ii) $m = \dfrac{-1-5}{5-2} = -2$
$y - 6 = \dfrac{-1}{m}(x-8)$
$x - 2y + 4 = 0$

Q42

(i) $2(x-1)^2 - 1$ *OR* $a = 2, b = -1, c = -1$
$A = (1, -1)$

(ii) $2x^2 - 5x - 3 = 0 \Rightarrow (2x+1)(x-3) = 0$ **OE** in y
$x = -½,\ y = 3½$

(iii) Mid-point of $AP = (2, 3)$
Gradient of line = $\dfrac{\frac{1}{2}}{\frac{-5}{2}} = \dfrac{-1}{5}$
Equation is $y - 3 = \dfrac{-1}{5}(x-2)$ **OE**

Q43

(i) (2, 5) to (10, 9) gradient = ½
Equation of L_2 $y = \frac{1}{2}x$.
Gradient of perpendicular = -2
Eqn of Perp $y - 5 = -2(x-2)$
Sim Eqns → $C(3.6, 1.8)$

(ii) $d^2 = 1.6^2 + 3.2^2 \rightarrow d = 3.58$

Q44:

$\dfrac{x}{a} + \dfrac{y}{b} = 1$
$P(a, 0)$ and $Q(0, b)$
Distance → $\sqrt{(a^2 + b^2)} = \sqrt{45}$
Gradients → $\dfrac{-a}{b} = \dfrac{-1}{2}$
Solution of sim eqns → $a = 6, b = 3$

Q45

(i) $10^2 \sin 0.8 = 71.7$

(ii) sector(s) = (2) × ½ × 10^2 × 0.8 = (2) × 40
Total area = 80

(iii) arc(s) = (2) × 10 × 0.8
16+20 = 36

Q46

(i) $6x + 2 = 7\sqrt{x} \Rightarrow 6\left(\sqrt{x}\right)^2 - 7\sqrt{x} + 2 = 0$
$\left(3\sqrt{x} - 2\right)\left(2\sqrt{x} - 1\right) = 0$
$\sqrt{x} = \dfrac{2}{3}$ or $\dfrac{1}{2}$
$x = \dfrac{4}{9}$ or $\dfrac{1}{4}$ (or 0.444, 0.25)

OR $(6x+2)^2 = 49x \rightarrow 36x^2 - 25x + 4 = 0$
$(9x-4)(4x-1) = 0$
$x = \dfrac{4}{9}$ or $\dfrac{1}{4}$ (or 0.444, 0.25) oe

(ii) $7^2 - 4 \times 6 \times k (= 0)$
$k = \dfrac{49}{24}$ or 2.04

OR $\dfrac{d}{dx}\left(7x^{\frac{1}{2}}\right) = \dfrac{d}{dx}(6x + k) \rightarrow \dfrac{7}{2}x^{\frac{-1}{2}} = 6$
$x = \dfrac{49}{144},\ y = \dfrac{49}{12} \rightarrow k = \dfrac{49}{24}$ or 2.04