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Trigonometry P1

Q1

Find all the values of x in the interval $0^\circ \leqslant x \leqslant 180^\circ$ which satisfy the equation $\sin 3x + 2\cos 3x = 0$. [4]

Q2

(i) Show that the equation $\sin^2\theta + 3\sin\theta\cos\theta = 4\cos^2\theta$ can be written as a quadratic equation in $\tan\theta$. [2]

(ii) Hence, or otherwise, solve the equation in part **(i)** for $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

Q3

(i) Show that the equation $4\sin^4\theta + 5 = 7\cos^2\theta$ may be written in the form $4x^2 + 7x - 2 = 0$, where $x = \sin^2\theta$. [1]

(ii) Hence solve the equation $4\sin^4\theta + 5 = 7\cos^2\theta$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [4]

Q4

(i) Sketch and label, on the same diagram, the graphs of $y = 2\sin x$ and $y = \cos 2x$, for the interval $0 \leqslant x \leqslant \pi$. [4]

(ii) Hence state the number of solutions of the equation $2\sin x = \cos 2x$ in the interval $0 \leqslant x \leqslant \pi$. [1]

Q5

(i) Show that the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$ can be expressed as $\tan\theta = 3$. [2]

(ii) Hence solve the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [2]

Q6

Solve the equation $3\sin^2\theta - 2\cos\theta - 3 = 0$, for $0^\circ \leqslant \theta \leqslant 180^\circ$. [4]

Q7

Solve the equation

$$\sin 2x + 3\cos 2x = 0,$$

for $0^\circ \leqslant x \leqslant 180^\circ$. [4]

Q8

Given that $x = \sin^{-1}\left(\frac{2}{5}\right)$, find the exact value of

(i) $\cos^2 x$, [2]

(ii) $\tan^2 x$. [2]

Q9

Prove the identity $\dfrac{1 - \tan^2 x}{1 + \tan^2 x} \equiv 1 - 2\sin^2 x$. [4]

Q10

(i) Show that the equation $3\sin x\tan x = 8$ can be written as $3\cos^2 x + 8\cos x - 3 = 0$. [3]

(ii) Hence solve the equation $3\sin x\tan x = 8$ for $0^\circ \leqslant x \leqslant 360^\circ$. [3]

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Q11

(i) Sketch the graph of the curve $y = 3\sin x$, for $-\pi \leqslant x \leqslant \pi$. [2]

The straight line $y = kx$, where k is a constant, passes through the maximum point of this curve for $-\pi \leqslant x \leqslant \pi$.

(ii) Find the value of k in terms of π. [2]

(iii) State the coordinates of the other point, apart from the origin, where the line and the curve intersect. [1]

Q12

(i) Show that the equation $2\tan^2\theta\cos\theta = 3$ can be written in the form $2\cos^2\theta + 3\cos\theta - 2 = 0$. [2]

(ii) Hence solve the equation $2\tan^2\theta\cos\theta = 3$, for $0° \leqslant \theta \leqslant 360°$. [3]

Q13

Prove the identity

$$\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} \equiv \frac{2}{\cos x}.$$ [4]

Q14

The function f is such that $f(x) = a - b\cos x$ for $0° \leqslant x \leqslant 360°$, where a and b are positive constants. The maximum value of $f(x)$ is 10 and the minimum value is -2.

(i) Find the values of a and b. [3]

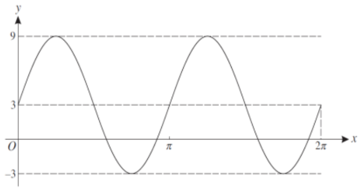
(ii) Solve the equation $f(x) = 0$. [3]

(iii) Sketch the graph of $y = f(x)$. [2]

Q15

Prove the identity $\dfrac{\sin x}{1-\sin x} - \dfrac{\sin x}{1+\sin x} \equiv 2\tan^2 x$. [3]

Q16



The diagram shows the graph of $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$.

(i) Find the values of a, b and c. [3]

(ii) Find the smallest value of x in the interval $0 \leqslant x \leqslant 2\pi$ for which $y = 0$. [3]

Q17

Solve the equation $3\tan(2x + 15°) = 4$ for $0° \leqslant x \leqslant 180°$. [4]

Q18

The equation of a curve is $y = 3\cos 2x$. The equation of a line is $x + 2y = \pi$. On the same diagram, sketch the curve and the line for $0 \leqslant x \leqslant \pi$. [4]

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Q19

The function f is defined by $f : x \mapsto 5 - 3\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [3]

(iii) State, with a reason, whether f has an inverse. [1]

Q20

(i) Prove the identity $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$. [3]

(ii) Solve the equation $(\sin x + \cos x)(1 - \sin x \cos x) = 9\sin^3 x$ for $0° \leqslant x \leqslant 360°$. [3]

Q21

The acute angle x radians is such that $\tan x = k$, where k is a positive constant. Express, in terms of k,

(i) $\tan(\pi - x)$, [1]

(ii) $\tan(\frac{1}{2}\pi - x)$, [1]

(iii) $\sin x$. [2]

Q22

The function f is such that $f(x) = 2\sin^2 x - 3\cos^2 x$ for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a + b\cos^2 x$, stating the values of a and b. [2]

(ii) State the greatest and least values of $f(x)$. [2]

(iii) Solve the equation $f(x) + 1 = 0$. [3]

Q23

(i) Show that the equation

$$3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$$

can be written in the form $\tan x = -\frac{3}{4}$. [2]

(ii) Solve the equation $3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$, for $0° \leqslant x \leqslant 360°$. [2]

Q24

The function $f : x \mapsto a + b\cos x$ is defined for $0 \leqslant x \leqslant 2\pi$. Given that $f(0) = 10$ and that $f\left(\frac{2}{3}\pi\right) = 1$, find

(i) the values of a and b, [2]

(ii) the range of f, [1]

(iii) the exact value of $f\left(\frac{5}{6}\pi\right)$. [2]

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Q25

(i) Show that the equation $2\sin x \tan x + 3 = 0$ can be expressed as $2\cos^2 x - 3\cos x - 2 = 0$. [2]

(ii) Solve the equation $2\sin x \tan x + 3 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

Q26

(i) Prove the identity $\dfrac{\sin x \tan x}{1 - \cos x} \equiv 1 + \dfrac{1}{\cos x}$. [3]

(ii) Hence solve the equation $\dfrac{\sin x \tan x}{1 - \cos x} + 2 = 0$, for $0° \leqslant x \leqslant 360°$. [3]

Q27

Prove the identity

$$\tan^2 x - \sin^2 x \equiv \tan^2 x \sin^2 x.$$ [4]

Q28

Solve the equation $15\sin^2 x = 13 + \cos x$ for $0° \leqslant x \leqslant 180°$. [4]

Q29

(i) Sketch the curve $y = 2\sin x$ for $0 \leqslant x \leqslant 2\pi$. [1]

(ii) By adding a suitable straight line to your sketch, determine the number of real roots of the equation

$$2\pi \sin x = \pi - x.$$

State the equation of the straight line. [3]

Q30

(i) Show that the equation $2\tan^2\theta \sin^2\theta = 1$ can be written in the form

$$2\sin^4\theta + \sin^2\theta - 1 = 0.$$ [2]

(ii) Hence solve the equation $2\tan^2\theta \sin^2\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

Q31

(i) Prove the identity $\dfrac{\cos\theta}{\tan\theta(1 - \sin\theta)} \equiv 1 + \dfrac{1}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\cos\theta}{\tan\theta(1 - \sin\theta)} = 4$, for $0° \leqslant \theta \leqslant 360°$. [3]

Q32

The function f is such that $f(x) = 3 - 4\cos^k x$, for $0 \leqslant x \leqslant \pi$, where k is a constant.

(i) In the case where $k = 2$,

(a) find the range of f, [2]

(b) find the exact solutions of the equation $f(x) = 1$. [3]

(ii) In the case where $k = 1$,

(a) sketch the graph of $y = f(x)$, [2]

(b) state, with a reason, whether f has an inverse. [1]

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Q33

(i) Prove the identity $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0° \leqslant \theta \leqslant 360°$. [4]

Q34

(i) Sketch, on a single diagram, the graphs of $y = \cos 2\theta$ and $y = \frac{1}{2}$ for $0 \leqslant \theta \leqslant 2\pi$. [3]

(ii) Write down the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $0 \leqslant \theta \leqslant 2\pi$. [1]

(iii) Deduce the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $10\pi \leqslant \theta \leqslant 20\pi$. [1]

Q35

(i) Sketch, on the same diagram, the graphs of $y = \sin x$ and $y = \cos 2x$ for $0° \leqslant x \leqslant 180°$. [3]

(ii) Verify that $x = 30°$ is a root of the equation $\sin x = \cos 2x$, and state the other root of this equation for which $0° \leqslant x \leqslant 180°$. [2]

(iii) Hence state the set of values of x, for $0° \leqslant x \leqslant 180°$, for which $\sin x < \cos 2x$. [2]

Q36

(i) Given that

$$3\sin^2 x - 8\cos x - 7 = 0,$$

show that, for real values of x,

$$\cos x = -\tfrac{2}{3}.$$ [3]

(ii) Hence solve the equation

$$3\sin^2(\theta + 70°) - 8\cos(\theta + 70°) - 7 = 0$$

for $0° \leqslant \theta \leqslant 180°$. [4]

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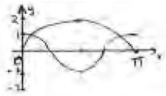
Answers:

Q1: 38.9,158.9, 98.9

Q2: $\theta = 45° \text{ or } 104°$

Q3: (ii) $\theta = 30°, 150°, 210° \;\&\; 330°$

Q4 **4 (i)**



(ii) → 2 points of intersection.

Q5 (i) s + c = 2s – 2c → s = 3c
→ tan θ = 3

(ii) → θ = 71.6° or 251.6°

Q6 (i) $3\sin^2\theta - 2\cos\theta - 3 = 0$
Use of $s^2 + c^2 = 1$
$3\cos^2\theta + 2\cos\theta = 0$

$\cos\theta = 0, \quad \theta = 90°$

or $\cos\theta = -2/3, \quad \theta = 131.8°$

Q7:

tan 2x = –3
2x = 180 – 71.6
or 2x = 360 – 71.6

→ x = 54.2° or 144.2°

Q8

$x = \sin^{-1}\frac{2}{5} \rightarrow \sin x = \frac{2}{5}$

(i) $\cos^2 x = 1 - \sin^2 x = \frac{21}{25}$

(ii) $\tan^2 x = \dfrac{\sin^2 x}{\cos^2 x} = \frac{4}{21}$

Q9

Use of $t = s/c$
→ $(c^2 - s^2) \div (c^2 + s^2)$
Use of $c^2 + s^2 = 1$
→ $(c^2 - s^2)$ → $1 - 2\sin^2 x$

Q10

(i) $3\sin x \tan x = 8$
Uses tan = sine ÷ cosine
Uses sin² = 1 – cos ²
→ $3\cos^2 x + 8\cos - 3 = 0$

(ii) (3c – 1)(c + 3) =0 or formula
→ cosx = ⅓ as only solution.
x = 70.5° or 289.5° only.

Q11: (i) x=π/2, y=3, k=6/π (ii) (-π/2,-3)

Q12:

(i) $2\tan^2\theta\cos\theta = 3$
Replaces $\tan^2\theta$ by $\dfrac{\sin^2\theta}{\cos^2\theta}$ and
Replaces $\sin^2\theta$ by $1 - \cos^2\theta$

→ $2\cos^2\theta + 3\cos\theta - 2 = 0$

(ii) Soln of quadratic → ½ and –2

→ 60° and 300°

Q13

$\dfrac{1+\sin x}{\cos x} + \dfrac{\cos x}{1+\sin x} \equiv \dfrac{2}{\cos x}$

LHS $\dfrac{(1+\sin x)^2 + \cos^2 x}{\cos x(1+\sin x)}$

$= \dfrac{2 + 2\sin x}{\cos x(1+\sin x)}$

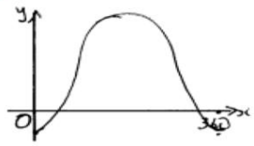
$= \dfrac{2}{\cos x}$

Q14

$x \mapsto a - b\cos x$
(i) $a + b = 10$ and $a - b = -2$
→ $a = 4$ and $b = 6$

(ii) $4 - 6\cos x = 0$
→ $\cos x = 2/3$
→ $x = 48.2°$ or $311.8°$

(iii)



Q15

$\dfrac{s}{1-s} - \dfrac{s}{1+s} = \dfrac{2s^2}{1-s^2}$

Use of $1 - s^2 = c^2$
→ $\dfrac{2s^2}{c^2}$
→ $2t^2$

Q16:

(i) $a = 6$
$b = 2$
$c = 3$

(ii) $6\sin 2x + 3 = 0$
→ $\sin 2x = -½$
Works with "2x" first
$x = \dfrac{7\pi}{12}$ or 1.83.

Q17 $3\tan(2x + 15°) = 4$
$\tan(2x + 15°) = 1⅓$
Sets the bracket to $\tan^{-1}(1⅓)$
$2x + 15 = 53.13°$ or $233.13°$
→ $x = 19.1°$ or $109.1°$

Q18 SKETCH

Q19

(i) $2 \leqslant f(x) \leqslant 8$

(ii) $x \mapsto 5 - 3\sin 2x$

(iii) No inverse – not 1 : 1.

Q20:

(i) $(\sin x + \cos x)(1 - \sin x\cos x)$
$= \sin x + \cos x - \sin^2 x\cos x - \cos^2 x\sin x$
$\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$
→ $\sin^3 x + \cos^3 x$

(ii)
$(\sin x + \cos x)(1 - \sin x\cos x) = 9\sin^3 x$
Uses part (i) → $8\sin^3 x = \cos^3 x$
→ $\tan^3 x = \frac{1}{8}$ → $\tan x = ½$
→ $x = 26.6°$ and $206.6°$

Q21

$\tan x = k$

(i) $\tan(\pi - x) = -k$

(ii) $\tan\left(\dfrac{\pi}{2} - x\right) = \dfrac{1}{k}$

(iii) $\sin x = \dfrac{k}{\sqrt{1+k^2}}$ from 90° triangle.

Q22

$x \mapsto 2\sin^2 x - 3\cos^2 x$

(i) $2(1 - \cos^2 x) - 3\cos^2 x$
→ $2 - 5\cos^2 x \quad (a = 2, b = -5)$

(ii) Values are –3 and 2

(iii) $2 - 5\cos^2 x = -1$
→ $\cos^2 x = 0.6$
$x = 0.685, 2.46$ (accept 0.684)

Q23

(i) $3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$
→ $6s - 3c = 2s - 6c$ → $4s = -3c$
→ $\tan x = -\frac{3}{4}$

(ii) $x = 180 - 36.9 = 143.1°$ or
$x = 360 - 36.9 = 323.1°$

Q24:

$f: x \mapsto a + b\cos x$

(i) $f(0) = 10, \; a + b = 10$
$f(^2/_3\pi) = 1, \; a - \dfrac{b}{2} = 1$
→ $a = 4, b = 6$

(ii) Range is –2 to 10.

(iii) $\cos\left(\dfrac{5}{6}\pi\right) = -\cos\left(\dfrac{1}{6}\pi\right) = -\dfrac{\sqrt{3}}{2}$

→ $4 - 3\sqrt{3}$

Q25 (i) $2\sin x\tan x + 3 = 0$

$2\sin x\dfrac{\sin x}{\cos x} + 3 = 0$

$2\dfrac{\left(1 - \cos^2 x\right)}{\cos x} + 3 = 0$

→ $2\cos^2 x - 3\cos x - 2 = 0$

(ii) $2\cos^2 x - 3\cos x - 2 = 0$
→ $\cos x = -½$ or 2
$x = 120°$ or $240°$

Q26 (i) $\dfrac{\sin x\tan x}{1 - \cos x} = \dfrac{\sin^2 x}{\cos x(1 - \cos x)}$

$= \dfrac{1 - \cos^2 x}{\cos x(1 - \cos x)}$

$= \dfrac{(1 - \cos x)(1 + \cos x)}{\cos x(1 - \cos x)} = \dfrac{1}{\cos x} + 1$

(ii) $\dfrac{1}{\cos x} + 1 + 2 = 0$
→ $\cos x = -⅓$
→ $x = 109.5°$ or $250.5°$

Q27

LHS = $\sin^2 x/\cos^2 x - \sin^2 x$
$\sin^2 x(1 - \cos^2 x)/\cos^2 x$
$\dfrac{\sin^2 x\sin^2 x}{\cos^2 x}$ oe
$\tan^2 x\sin^2 x$

OR RHS = $\dfrac{\sin^2 x}{\cos^2 x} \cdot \sin^2 x$

$\sin^2 x(1 - \cos^2 x)/\cos^2 x$
$(\sin^2 x/\cos^2 x) - \sin^2 x$
$\tan^2 x - \sin^2 x$

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Q28:

$15\cos^2 x + \cos x - 2 = 0$

$(5\cos x + 2)(3\cos x - 1) = 0$

113(.6), 70.5

Q29

(i) Correct sine curve

(ii) Required line $y = 1 - \frac{x}{\pi}$

Line through (0, 1), (π, 0) drawn

3 roots

Q30

(i) $\frac{2\sin^2\theta\sin^2\theta}{1-\sin^2\theta} = 1$

$2\sin^4\theta + \sin^2\theta - 1 = 0$

(ii) $(2\sin^2\theta - 1)(\sin^2\theta + 1) = 0$

$\sin\theta = \frac{(\pm)1}{\sqrt{2}}$

$\theta = 45°, 135°$

$\theta = 225°, 315°$

Q31

(i) $\frac{\cos\theta}{\tan\theta(1-\sin\theta)} \equiv \frac{\cos^2\theta}{\sin\theta(1-\sin\theta)}$

$= \frac{1-\sin^2\theta}{\sin\theta(1-\sin\theta)}$

$= \frac{1+\sin\theta}{\sin\theta} = \frac{1}{\sin\theta} + 1$

(ii) $\frac{\cos\theta}{\tan\theta(1-\sin\theta)} = 4 \rightarrow \frac{1}{\sin\theta} + 1 = 4$

$\rightarrow \sin\theta = ⅓ \rightarrow \theta = 19.5°, 160.5°$

Q32:

(i) **(a)** $f(x) = 3 - 4\cos^2 x$.
One limit is -1
Other limit is 3

(b) $3 - 4\cos^2 x = 1 \rightarrow \cos^2 x = ½$

$\rightarrow \cos x = \pm\frac{1}{\sqrt{2}}$

$\rightarrow x = ¼\pi$ or $¾\pi$

(ii) **(a)**

(b) f has an inverse since it is 1:1 or increasing or no turning points.

Q33

(i) $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$

$\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta} = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{2}{5}$

$\frac{1-\cos\theta}{1+\cos\theta} = \frac{2}{5}$

$\cos\theta = \frac{3}{7}$

$\theta = 64.6°$ or $295.4°$

Q34

(i) Correct cosine curve for at least 1 oscillation

Exactly 2 complete oscillations in $[0, 2\pi]$

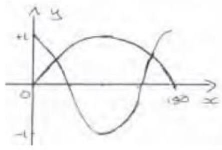
Line $y = \frac{1}{2}$ correct

(ii) 4

(iii) 20

Q35

(i)



(ii) Evidence of sin 30 = cos 60 = 0.5
Other root is 150°

(iii) $0 \le x < 30$ and $150 < x \le 180$
($x < 30$ or $x > 150$ ok)

Q36:

(i) $3\cos^2 x + 8\cos x + 4 = 0$

$(3\cos x + 2)(\cos x + 2) = 0$

$\cos x = -\frac{2}{3}$

(ii) $\cos(\theta + 70) = -\frac{2}{3}$, $\theta = 61.8$

$\theta + 70 = 131.8$ (or 228.2)

$\theta = 158.2$