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Vectors P1

Q1

The points A, B, C and D have position vectors $3\mathbf{i} + 2\mathbf{k}$, $2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $2\mathbf{j} + 7\mathbf{k}$ and $-2\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$ respectively.

(i) Use a scalar product to show that BA and BC are perpendicular. [4]

(ii) Show that BC and AD are parallel and find the ratio of the length of BC to the length of AD. [4]

Q2

Relative to an origin O, the position vectors of the points A, B, C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} -1 \\ 0 \\ q \end{pmatrix},$$

where p and q are constants. Find

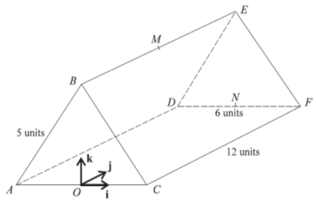
(i) the unit vector in the direction of $\overrightarrow{AB}$, [3]

(ii) the value of p for which angle $AOC = 90°$, [3]

(iii) the values of q for which the length of $\overrightarrow{AD}$ is 7 units. [4]

Q3

The diagram shows a triangular prism with a horizontal rectangular base $ADFC$, where $CF = 12$ units and $DF = 6$ units. The vertical ends ABC and DEF are isosceles triangles with $AB = BC = 5$ units. The mid-points of BE and DF are M and N respectively. The origin O is at the mid-point of AC.



Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OC, ON and OB respectively.

(i) Find the length of OB. [1]

(ii) Express each of the vectors $\overrightarrow{MC}$ and $\overrightarrow{MN}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(iii) Evaluate $\overrightarrow{MC}.\overrightarrow{MN}$ and hence find angle CMN, giving your answer correct to the nearest degree. [4]

Q4

The points A and B have position vectors $\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ and $-5\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ respectively, relative to an origin O.

(i) Use a scalar product to calculate angle AOB, giving your answer in radians correct to 3 significant figures. [4]

(ii) The point C is such that $\overrightarrow{AB} = 2\overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

Q5

Relative to an origin O, the position vectors of points P and Q are given by

$$\overrightarrow{OP} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OQ} = \begin{pmatrix} 2 \\ 1 \\ q \end{pmatrix},$$

where q is a constant.

(i) In the case where $q = 3$, use a scalar product to show that $\cos POQ = \frac{1}{7}$. [3]

(ii) Find the values of q for which the length of $\overrightarrow{PQ}$ is 6 units. [4]

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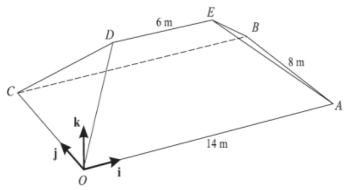
Q6

The diagram shows the roof of a house. The base of the roof, $OABC$, is rectangular and horizontal with $OA = CB = 14$ m and $OC = AB = 8$ m. The top of the roof DE is 5 m above the base and $DE = 6$ m. The sloping edges OD, CD, AE and BE are all equal in length.



Unit vectors **i** and **j** are parallel to OA and OC respectively and the unit vector **k** is vertically upwards.

(i) Express the vector $\overrightarrow{OD}$ in terms of **i**, **j** and **k**, and find its magnitude. [4]

(ii) Use a scalar product to find angle DOB. [4]

Q7

The position vectors of points A and B are $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ respectively, relative to an origin O.

(i) Calculate angle AOB. [3]

(ii) The point C is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

Q8

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}.$$

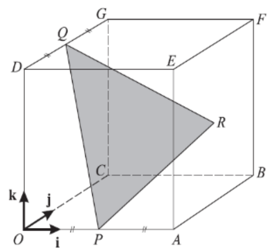
(i) Given that C is the point such that $\overrightarrow{AC} = 2\overrightarrow{AB}$, find the unit vector in the direction of $\overrightarrow{OC}$. [4]

The position vector of the point D is given by $\overrightarrow{OD} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$, where k is a constant, and it is given that $\overrightarrow{OD} = m\overrightarrow{OA} + n\overrightarrow{OB}$, where m and n are constants.

(ii) Find the values of m, n and k. [4]

Q9

The diagram shows a cube $OABCDEFG$ in which the length of each side is 4 units. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The mid-points of OA and DG are P and Q respectively and R is the centre of the square face $ABFE$.



(i) Express each of the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ in terms of **i**, **j** and **k**. [3]

(ii) Use a scalar product to find angle QPR. [4]

(iii) Find the perimeter of triangle PQR, giving your answer correct to 1 decimal place. [3]

Q10

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

(i) Use a scalar product to find angle AOB, correct to the nearest degree. [4]

(ii) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

(iii) The point C is such that $\overrightarrow{OC} = 6\mathbf{j} + p\mathbf{k}$, where p is a constant. Given that the lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are equal, find the possible values of p. [4]

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Q11

Relative to an origin O, the position vectors of points A and B are $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}$ respectively.

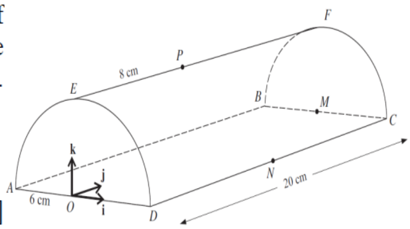
(i) Find the value of p for which OA and OB are perpendicular. [2]

(ii) In the case where $p = 6$, use a scalar product to find angle AOB, correct to the nearest degree. [3]

(iii) Express the vector $\overrightarrow{AB}$ is terms of p and hence find the values of p for which the length of AB is 3.5 units. [4]

Q12

The diagram shows a semicircular prism with a horizontal rectangular base $ABCD$. The vertical ends AED and BFC are semicircles of radius 6 cm. The length of the prism is 20 cm. The mid-point of AD is the origin O, the mid-point of BC is M and the mid-point of DC is N. The points E and F are the highest points of the semicircular ends of the prism. The point P lies on EF such that $EP = 8$ cm.



Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OD, OM and OE respectively.

(i) Express each of the vectors $\overrightarrow{PA}$ and $\overrightarrow{PN}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(ii) Use a scalar product to calculate angle APN. [4]

Q13

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 8\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 7\mathbf{i} + 2\mathbf{j} - \mathbf{k}.$$

(i) Find the value of $\overrightarrow{OA}.\overrightarrow{OB}$ and hence state whether angle AOB is acute, obtuse or a right angle. [3]

(ii) The point X is such that $\overrightarrow{AX} = \frac{2}{5}\overrightarrow{AB}$. Find the unit vector in the direction of OX. [4]

Q14

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix}.$$

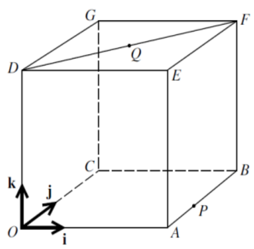
(i) Find angle AOB. [4]

(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and has magnitude 30. [3]

(iii) Find the value of the constant p for which $\overrightarrow{OA} + p\,\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OC}$. [3]

Q15

In the diagram, $OABCDEFG$ is a cube in which each side has length 6. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point P is such that $\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$ and the point Q is the mid-point of DF.



(i) Express each of the vectors $\overrightarrow{OQ}$ and $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(ii) Find the angle OQP. [4]

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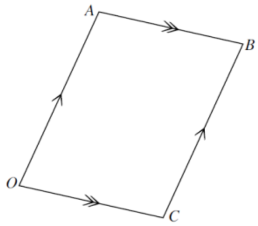
Q16

The diagram shows the parallelogram $OABC$. Given that $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$, find



(i) the unit vector in the direction of $\overrightarrow{OB}$, [3]

(ii) the acute angle between the diagonals of the parallelogram, [5]

(iii) the perimeter of the parallelogram, correct to 1 decimal place. [3]

Q17

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 1 \\ p \end{pmatrix}.$$

(i) Find the value of p for which $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$. [2]

(ii) Find the values of p for which the magnitude of $\overrightarrow{AB}$ is 7. [4]

Q18

Relative to an origin O, the position vectors of the points A, B and C are given by

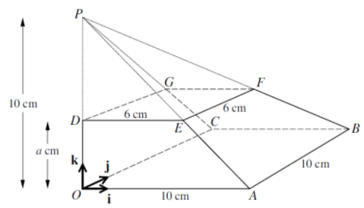
$$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}, \quad \overrightarrow{OC} = -\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}.$$

(i) Use a scalar product to find angle ABC. [6]

(ii) Find the perimeter of triangle ABC, giving your answer correct to 2 decimal places. [2]

Q19

The diagram shows a pyramid $OABCP$ in which the horizontal base $OABC$ is a square of side 10 cm and the vertex P is 10 cm vertically above O. The points D, E, F, G lie on OP, AP, BP, CP respectively and $DEFG$ is a horizontal square of side 6 cm. The height of $DEFG$ above the base is a cm. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and OD respectively.



(i) Show that $a = 4$. [2]

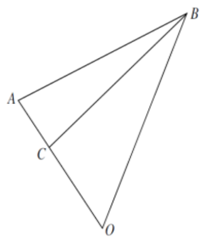
(ii) Express the vector $\overrightarrow{BG}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(iii) Use a scalar product to find angle GBA. [4]

Q20

The diagram shows triangle OAB, in which the position vectors of A and B with respect to O are given by

$$\overrightarrow{OA} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = -3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}.$$



C is a point on OA such that $\overrightarrow{OC} = p\overrightarrow{OA}$, where p is a constant.

(i) Find angle AOB. [4]

(ii) Find $\overrightarrow{BC}$ in terms of p and vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

(iii) Find the value of p given that BC is perpendicular to OA. [4]

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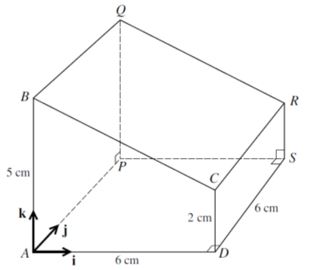
Q21

The diagram shows a prism $ABCDPQRS$ with a horizontal square base $APSD$ with sides of length 6 cm. The cross-section $ABCD$ is a trapezium and is such that the vertical edges AB and DC are of lengths 5 cm and 2 cm respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to AD, AP and AB respectively.



(i) Express each of the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(ii) Use a scalar product to calculate angle PCQ. [4]

Q22

Relative to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 10 \\ 0 \\ 6 \end{pmatrix}.$$

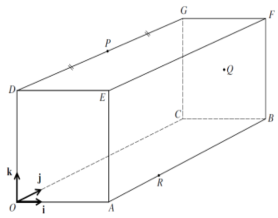
(i) Find angle ABC. [6]

The point D is such that $ABCD$ is a parallelogram.

(ii) Find the position vector of D. [2]

Q23

In the diagram, $OABCDEFG$ is a rectangular block in which $OA = OD = 6$ cm and $AB = 12$ cm. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point P is the mid-point of DG, Q is the centre of the square face $CBFG$ and R lies on AB such that $AR = 4$ cm.



(i) Express each of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(ii) Use a scalar product to find angle RQP. [4]

Q24

Relative to an origin O, the point A has position vector $4\mathbf{i} + 7\mathbf{j} - p\mathbf{k}$ and the point B has position vector $8\mathbf{i} - \mathbf{j} - p\mathbf{k}$, where p is a constant.

(i) Find $\overrightarrow{OA}.\overrightarrow{OB}$. [2]

(ii) Hence show that there are no real values of p for which OA and OB are perpendicular to each other. [1]

(iii) Find the values of p for which angle $AOB = 60°$. [4]

Q25

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k},$$

where p is a constant.

(i) Find the value of p for which angle AOB is $90°$. [3]

(ii) In the case where $p = 4$, find the vector which has magnitude 28 and is in the same direction as $\overrightarrow{AB}$. [4]

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Q26

Relative to an origin O, the position vectors of points A and B are $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ respectively.

(i) Use a scalar product to find angle BOA. [4]

The point C is the mid-point of AB. The point D is such that $\overrightarrow{OD} = 2\overrightarrow{OB}$.

(ii) Find $\overrightarrow{DC}$. [4]

Q27

Two vectors $\mathbf{u}$ and $\mathbf{v}$ are such that $\mathbf{u} = \begin{pmatrix} p^2 \\ -2 \\ 6 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ p-1 \\ 2p+1 \end{pmatrix}$, where p is a constant.

(i) Find the values of p for which $\mathbf{u}$ is perpendicular to $\mathbf{v}$. [3]

(ii) For the case where $p = 1$, find the angle between the directions of $\mathbf{u}$ and $\mathbf{v}$. [4]

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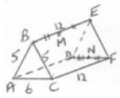
Answers:

Q1: 2:5

Q2: (i) $\frac{1}{6}(2i - 4j + 4k)$ (ii) $p = 10$ (iii) $q = 5$ & $q = -7$

Q3

7



(i) Height = 4

(ii) MC = 3i-6j-4k
MN = 6j – 4k

(iii) MC.MN = -36+16 = -20
MC.MN = $\sqrt{61}\sqrt{52}\cos\theta$
$\rightarrow \theta = 111°$

Q4

(i) (i + 7j + 2k).(–5i + 5j + 6k)
→ –5 + 35 + 12 = 42
42 = $\sqrt{54}\ \sqrt{86}\cos\theta$
→ angle AOB = 0.907

(ii) BC = ½ (b – a) = – 3i – j + 2k

OC = OB + BC = –5i + 5j + 6k – 3i – j + 2k = –8i + 4j + 8k

Unit Vector = (– 8i + 4j + 8k) ÷ 12

Q5:

$\overrightarrow{OP} = \begin{pmatrix}-2\\3\\1\end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix}2\\1\\q\end{pmatrix}$

(i) $\begin{pmatrix}-2\\3\\1\end{pmatrix}.\begin{pmatrix}2\\1\\q\end{pmatrix}$ with $q = 3$, $= -4+3+3 = 2$

$= \sqrt{14}.\sqrt{14}\cos\theta = 2$, $\cos\theta = \frac{1}{7}$

(ii) $\overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = \begin{pmatrix}4\\-2\\q-1\end{pmatrix}$

$16 + 4 + (q-1)^2 = 36$
$\rightarrow q = 5$ or $q = -3$

Q6

(i) Vector **OD = 4i + 4j + 5k**

Magnitude= $\sqrt{(4^2+4^2+5^2)} = \sqrt{57}$
→ Magnitude = 7.55m

(ii) Vector **OB = 14i + 8j**
OD.OB = 4×14 + 4×8 = 88
OD.OB = $\sqrt{57}.\sqrt{260}\cos\theta$
→ Angle DOB = 43.7°

Q7

$\mathbf{a} = \begin{pmatrix}-3\\6\\3\end{pmatrix}$ $\mathbf{b} = \begin{pmatrix}-1\\2\\4\end{pmatrix}$

(i) $\mathbf{a.b} = 3 + 12 + 12 = 27$
$\mathbf{a.b} = \sqrt{54}\times\sqrt{21}\cos\theta$
$\rightarrow \theta = 36.7°$ or 0.641 radians

(ii)

Vector $AB = \mathbf{b} - \mathbf{a} = \begin{pmatrix}2\\-4\\1\end{pmatrix}$

Vector $OC = \begin{pmatrix}-3\\6\\3\end{pmatrix} + 3\begin{pmatrix}2\\-4\\1\end{pmatrix} = \begin{pmatrix}3\\-6\\6\end{pmatrix}$

Unit vector = Vector OC ÷ 9
= ⅓i – ⅔ j + ⅔k

Q8:

$\overrightarrow{AB} = \begin{pmatrix}-1\\1\\-2\end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix}-2\\2\\-4\end{pmatrix}$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \begin{pmatrix}2\\3\\-6\end{pmatrix}$

Unit vector = $\frac{1}{7}\begin{pmatrix}2\\3\\-6\end{pmatrix}$

$m\begin{pmatrix}4\\1\\-2\end{pmatrix} + n\begin{pmatrix}3\\2\\-4\end{pmatrix} = \begin{pmatrix}1\\4\\k\end{pmatrix}$
$\rightarrow$ $4m + 3n = 1$ and $m + 2n = 4$
$\rightarrow$ $m = -2$ and $n = 3$
$\rightarrow$ $k = -8$

Q9

10 (i) $\overrightarrow{PR} = \begin{pmatrix}2\\2\\2\end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$

(ii) $\overrightarrow{PQ}.\overrightarrow{PR} = -4 + 4 + 8 = 8$
$|\overrightarrow{PQ}| = \sqrt{24}$ $|\overrightarrow{PR}| = \sqrt{12}$
$\overrightarrow{PQ}.\overrightarrow{PR} = \sqrt{12}\ \sqrt{24}\cos QPR$

Angle QPR = 61.9° or 1.08 rad

(iii) $\overrightarrow{QR} = \begin{pmatrix}4\\0\\-2\end{pmatrix}$ $|\overrightarrow{QR}| = \sqrt{20}$

Perimeter = $\sqrt{12} + \sqrt{24} + \sqrt{20}$ = 12.8 cm

Q10 $\overrightarrow{OA}$ = 2i + 3j – **k** $\overrightarrow{OB}$ = 4i – 3j + 2**k**

(i) $\overrightarrow{OA}.\overrightarrow{OB} = 8 - 9 - 2 = -3$
$\overrightarrow{OA}.\overrightarrow{OB} = \sqrt{14} \times \sqrt{29}\cos AOB$
$\rightarrow AOB = 99°$

(ii) $\overrightarrow{AB}$ = **b** – **a** = 2i – 6j + 3k
Magnitude of $\overrightarrow{AB} = \sqrt{49} = 7$
→ Unit vector = $\frac{1}{7}$ (2i - 6j + 3k)

(iii) $\overrightarrow{AC}$ = –2**i** + 3**j** +(p + 1)**k**
$4 + 9 + (p + 1)^2 = 49$
$\rightarrow p = 5$ or –7

Q11

OA = 2**i** + **j** + 2**k**, **OB** = 3**i** – 2**j** + p**k**

(i) (2**i** + **j** + 2**k**) . (3**i** – 2**j** + p**k**) = 0
$\rightarrow 6 - 2 + 2p = 0$
$\rightarrow p = -2$

(ii) (2**i** + **j** + 2**k**) . (3**i** – 2**j** + 6**k**)
$\rightarrow 6 - 2 + 12$ allow for ± this
$= \sqrt{9} \times \sqrt{49}\cos\theta$
$\rightarrow \theta = 40°$

(iii) **AB** = **i** – 3**j** + (p – 2)**k**
$1^2 + 3^2 + (p-2)^2 = 3.5^2$

$\rightarrow p = 0.5$ or 3.5

Q12:

(i) $\overrightarrow{PA} = -6\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$
$\overrightarrow{PN} = 6\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$

(ii) $\overrightarrow{PA}.\overrightarrow{PN} = -36 - 16 + 36 = -16$

$\cos APN = \dfrac{-16}{\sqrt{136}\sqrt{76}}$

$\rightarrow APN = 99°$

Q13

(i) **OA.OB** = 14 – 16 – 4 = –6
This is –ve → Obtuse angle.

(ii) **AB** = 5**i** + 10**j** – 5**k**
AX = $\frac{2}{5}$(**AB**)
OX = **OA** + **AX**
OX = 4**i** – 4**j** + 2**k**

Divides by the modulus
Unit vector = $\frac{1}{6}$(4**i** – 4**j** + 2**k**)

Q14

$\overrightarrow{OA} = \begin{pmatrix}2\\3\\-6\end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix}0\\-6\\8\end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix}-2\\5\\-2\end{pmatrix}$.

(i) Scalar product = –18 – 48
$-66 = |\mathbf{a}|\ |\mathbf{b}|\cos\theta$
$|\mathbf{a}| = 7$ and $|\mathbf{b}| = 10$
→ Angle AOB = 160.5°

(ii) $\overrightarrow{AC} = \mathbf{c} - \mathbf{a} = \begin{pmatrix}-4\\2\\4\end{pmatrix}$

Modulus = 6

Vector = $5 \times \begin{pmatrix}-4\\2\\4\end{pmatrix}$ or $\begin{pmatrix}-20\\10\\20\end{pmatrix}$

(iii) $\begin{pmatrix}2\\3-6p\\-6+8p\end{pmatrix}.\begin{pmatrix}-2\\5\\-2\end{pmatrix} = 0$

$\rightarrow p = \frac{1}{2}$

Q15

(i) $\overrightarrow{OQ} = 3\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$
$\overrightarrow{PQ} = -3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$

(ii) (3**i** + 3**j** + 6**k**).(–3**i** + **j** + 6**k**)
= –9 + 3 + 36 = 30
$30 = \sqrt{54}\sqrt{46}\cos\theta$
$\theta = 53.0°$

Cosine rule M1 modulus

M1 attempt at 3 sides

M1 A1 answer.

Q16:

$\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$.

(i) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC} = 4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$
Unit vector = $\frac{1}{6}$(4**i** + 2**j** + 4**k**)

(ii) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$
$\overrightarrow{AC}.\overrightarrow{OB} = 8 - 8 - 8 = -8$
$|\overrightarrow{OB}| = 6$; $|\overrightarrow{AC}| = \sqrt{24}$
$-8 = 6 \times \sqrt{24} \times \cos\theta$
$\theta = 105.8° \rightarrow 74.2°$

(iii) $OA = \sqrt{19}$ or $OC = \sqrt{11}$
Perimeter = 2(√ + √)
→ 15.4

Q17

(i) $-8 + 3 + p = 0$
$\rightarrow p = 5.$

(ii) Vector $\overrightarrow{AB}$ = **b** – **a**
= 6**i** – 2**j** + (p – 1) **k**

$36 + 4 + (p-1)^2 = 49$
$\rightarrow p = 4$ or $p = -2$

Q18

$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$,
$\overrightarrow{OC} = -\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}$

(i) (±) 2**i** + 4**j** + 4**k**
(±) 4**i** + 4**j** – 2**k**

$\overrightarrow{AB}.\overrightarrow{CB} = 16$
$\overrightarrow{AB}.\overrightarrow{CB} = \sqrt{36}\sqrt{36}\cos\theta$
$\theta = 63.6°$

(ii) Perimeter = $6 + 6 + \sqrt{40}$
or $6 + 6 + 6\sin 31.8° \times 2$
→ 18.32

Q19

(i) $\dfrac{10-a}{10} = \dfrac{6}{10}$ oe

$a = 4$

(ii) $\overrightarrow{BG} = -10\mathbf{j} - 10\mathbf{i} + 4\mathbf{k} + 6\mathbf{j}$
$= -10\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

(iii) $\overrightarrow{BG}.\overrightarrow{BA} = 40$

$\cos GBA = \dfrac{40}{\sqrt{132}\sqrt{100}}$

$GBA = 69.6°$

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Q20:

(i) $\overrightarrow{OA}.\overrightarrow{OB} = -6 + 2 + 12 = 8$

$\cos AOB = \dfrac{8}{\sqrt{14}\sqrt{29}}$

$AOB = 66.6°$

(ii) $3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + p(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$

(iii) $\overrightarrow{BC} = \mathbf{i}(3 + 2p) + \mathbf{j}(-2 + p) + \mathbf{k}(4 - 3p)$

Their $\overrightarrow{BC}.[2\mathbf{i} + \mathbf{j} - 3\mathbf{k}] = 0$

$2(3 + 2p) + (p - 2) - 3(4 - 3p) = 0$

$p = 4/7 \;\; 0.571$

Q21

(i) $\overrightarrow{CP} = -6\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$

$\overrightarrow{CQ} = -6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$

(ii) Scalar product = $36 + 36 - 6$

$66 = |\overrightarrow{CP}||\overrightarrow{CQ}| \cos\theta$

$|\overrightarrow{CP}| = \sqrt{76}, \;\; |\overrightarrow{CQ}| = \sqrt{81}$

Angle $PCQ = 32.7°$ (or 0.571 rad)

Q22:

(i) $\overrightarrow{BA}.\overrightarrow{BC}$ or $\overrightarrow{AB}.\overrightarrow{CB}$

$\overrightarrow{BA} = \begin{pmatrix}-2\\1\\2\end{pmatrix}, \;\; \overrightarrow{BC} = \begin{pmatrix}6\\-2\\3\end{pmatrix}$

$\overrightarrow{BA}.\overrightarrow{BC} = -8$

$= 3 \times 7 \times \cos\theta$

$\rightarrow \;\; \theta = 112.4°$ or 1.96 radians

(ii) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC}$

$= \begin{pmatrix}8\\1\\8\end{pmatrix}$

Q23

(i) $\overrightarrow{PQ} = 3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$

$\overrightarrow{RQ} = -3\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}$

(ii) $\overrightarrow{PQ}.\overrightarrow{RQ} = -9 + 48 - 9 = 30$

$= \sqrt{54}\,\sqrt{82}\cos RQP$

$\rightarrow RQP = 63.2°$

Q24

(i) $(4i + 7j - pk).(8i - j - pk) = 25 + p^2$

(ii) $25 + p^2 = 0 \Rightarrow$ no real solutions

(iii) $\cos 60 = \dfrac{OA.OB}{|OA||OB|}$ used

$|OA| = \sqrt{65 + p^2}$ or $|OB| = \sqrt{65 + p^2}$

$\dfrac{25 + p^2}{65 + p^2} = \dfrac{1}{2}$ or $\dfrac{his\;scalar\,(i)}{65 + p^2} = \dfrac{1}{2}$

$p = \pm 3.87$ or $\pm\sqrt{15}$

Q25:

$\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k}, \overrightarrow{OB} = 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k}$

(i) $\overrightarrow{OA}.\overrightarrow{OB} = 10 + 7 + 2p$

$= 0 \;\;\rightarrow p = -8½$

(ii) $\mathbf{AB} = -3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$

Modulus = $\sqrt{(9+36+4)}$

Magnitude 28 $\rightarrow$ 28 ×unit vector

$\rightarrow -12\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}$.

Q26

(i) Scalar product = 15−8+3

$10 = |\mathbf{OA}|\,|\mathbf{OB}| \cos\theta$

$|\mathbf{OA}| = \sqrt{26}, \;\; |\mathbf{OB}| = \sqrt{38}$

Angle BOA = 71.4 or 71.5

or 1.25 radians

(ii) $\mathbf{a}+½(\mathbf{b}-\mathbf{a})$ or $\mathbf{b}+½(\mathbf{a}-\mathbf{b})$ or $½(\mathbf{a}+\mathbf{b})$

$-2\mathbf{b}$ + *their* $\mathbf{c}$ oe

$-6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$

Q27:

(i) $2p^2 - 2p + 2 + 12p + 6 \rightarrow 2p^2 + 10p + 8$

$\mathbf{u}.\mathbf{v} = 0$

$(p+1)(p+4) = 0 \rightarrow p = -1$ **or** $p = -4$

(ii) $\mathbf{u}.\mathbf{v} = 2 + 0 + 18 = 20$

$|\mathbf{u}| = \sqrt{41}$ or $|\mathbf{v}| = \sqrt{13}$

$20 = \sqrt{41} \times \sqrt{13} \times \cos\theta$ oe

$\theta = 30.0°$ or 0.523 rads