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Binomial Theorem P1

Q1

Find the value of the coefficient of $\frac{1}{x}$ in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [3]

Q2

Find the coefficient of x^3 in the expansion of

(i) $(1 + 2x)^6$, [3]

(ii) $(1 - 3x)(1 + 2x)^6$. [3]

Q3

Find the coefficient of x in the expansion of $\left(3x - \frac{2}{x}\right)^5$. [4]

Q4

(i) Find the first 3 terms in the expansion of $(2 - x)^6$ in ascending powers of x. [3]

(ii) Find the value of k for which there is no term in x^2 in the expansion of $(1 + kx)(2 - x)^6$. [2]

Q5

The first three terms in the expansion of $(2 + ax)^n$, in ascending powers of x, are $32 - 40x + bx^2$. Find the values of the constants n, a and b. [5]

Q6

Find the coefficient of x^2 in the expansion of $\left(x + \frac{2}{x}\right)^6$. [3]

Q7

(i) Find the first three terms in the expansion of $(2 + u)^5$ in ascending powers of u. [3]

(ii) Use the substitution $u = x + x^2$ in your answer to part **(i)** to find the coefficient of x^2 in the expansion of $(2 + x + x^2)^5$. [2]

Q8

(i) Find the first 3 terms in the expansion, in ascending powers of x, of $(2 + x^2)^5$. [3]

(ii) Hence find the coefficient of x^4 in the expansion of $(1 + x^2)^2(2 + x^2)^5$. [3]

Q9

Find the value of the coefficient of x^2 in the expansion of $\left(\frac{x}{2} + \frac{2}{x}\right)^6$. [3]

Q10

(i) Find the first 3 terms in the expansion of $(2 + 3x)^5$ in ascending powers of x. [3]

(ii) Hence find the value of the constant a for which there is no term in x^2 in the expansion of $(1 + ax)(2 + 3x)^5$. [2]

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Q11

(i) Find the first 3 terms in the expansion of $(2-x)^6$ in ascending powers of x. [3]

(ii) Given that the coefficient of x^2 in the expansion of $(1+2x+ax^2)(2-x)^6$ is 48, find the value of the constant a. [3]

Q12

(i) Find, in terms of the non-zero constant k, the first 4 terms in the expansion of $(k+x)^8$ in ascending powers of x. [3]

(ii) Given that the coefficients of x^2 and x^3 in this expansion are equal, find the value of k. [2]

Q13

(i) Find the first 3 terms in the expansion of $\left(2x-\frac{3}{x}\right)^5$ in descending powers of x. [3]

(ii) Hence find the coefficient of x in the expansion of $\left(1+\frac{2}{x^2}\right)\left(2x-\frac{3}{x}\right)^5$. [2]

Q14

(i) Find the first 3 terms in the expansion of $(1+ax)^5$ in ascending powers of x. [2]

(ii) Given that there is no term in x in the expansion of $(1-2x)(1+ax)^5$, find the value of the constant a. [2]

(iii) For this value of a, find the coefficient of x^2 in the expansion of $(1-2x)(1+ax)^5$. [3]

Q15

(i) Find the first three terms, in descending powers of x, in the expansion of $\left(x-\frac{2}{x}\right)^6$. [3]

(ii) Find the coefficient of x^4 in the expansion of $(1+x^2)\left(x-\frac{2}{x}\right)^6$. [2]

Q16

In the expansion of $(1+ax)^6$, where a is a constant, the coefficient of x is -30. Find the coefficient of x^3. [4]

Q17

(i) Find the first 3 terms in the expansion, in ascending powers of x, of $(1-2x^2)^8$. [2]

(ii) Find the coefficient of x^4 in the expansion of $(2-x^2)(1-2x^2)^8$. [2]

Q18

Find the term independent of x in the expansion of $\left(x-\frac{1}{x^2}\right)^9$. [3]

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Q19

Find the coefficient of x in the expansion of $\left(x + \frac{2}{x^2}\right)^7$. [3]

Q20

(i) Find the terms in x^2 and x^3 in the expansion of $\left(1 - \frac{3}{2}x\right)^6$. [3]

(ii) Given that there is no term in x^3 in the expansion of $(k + 2x)\left(1 - \frac{3}{2}x\right)^6$, find the value of the constant k. [2]

Q21

The coefficient of x^3 in the expansion of $(a + x)^5 + (1 - 2x)^6$, where a is positive, is 90. Find the value of a. [5]

Q22

Find the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^6$. [3]

Q23

(i) Find the first 3 terms in the expansion of $(2 - y)^5$ in ascending powers of y. [2]

(ii) Use the result in part **(i)** to find the coefficient of x^2 in the expansion of $\left(2 - (2x - x^2)\right)^5$. [3]

Q24

The coefficient of x^2 in the expansion of $\left(k + \frac{1}{3}x\right)^5$ is 30. Find the value of the constant k. [3]

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Answers

Q1: -40

Q2: (i)160 (ii) -20

Q3: $(3x-2/x)^5$
Required term has ${}_5C_2$ or ${}_5C_3 = 10$
Also has 3^3 and 2^2
→ 1080

Q4
(i) $(2-x)^6 = 64 - 192x + 240x^2$

(ii) $(1+kx)(2-x)^6$
coeff of $x^2 = 240 - 192k$
$= 0 \rightarrow k = 5/4$ or 1.25

Q5
$(2+ax)^n$
1st term = $2^n = 32 \rightarrow n = 5$
2nd term = $n.2^{n-1}(ax) = -40x$
3rd term = $n(n-1)\frac{1}{2}.2^{n-2}(ax)^2$
$\rightarrow a = -\frac{1}{2}$
$\rightarrow b = 20$

Q6
1. $\left(x+\frac{2}{x}\right)^6$ Term in x^2 has
${}_6C_2$ - needs factorials or 15.
$* (x)^4 * (2/x)^2$
→ 60 (needs selecting)
(first 2 marks can be obtained from expansion only)

Q7:
(i) $(2+u)^5 = 32 + 80u + 80u^2$

(ii) ... $80(x+x^2)+80(x+x^2)^2$
→ coeff of x^2 of 80 + 80 = 160

Q8
(i) $(2+x^2)^5 = 2^5 + 5.2^4.x^2 + 10.2^3.x^4$
→ $32 + 80x^2 + 80x^4$
(allow 2^5 for 32)

(ii) $(1+x^2)^2 = 1 + 2x^2 + x^4$
Product has 3 terms in x^4
→ 80 + 160 + 32 = 272

Q9
$\left(\frac{x}{2}+\frac{2}{x}\right)^6$
Term in $x^2\left(\frac{x}{2}\right)^4\left(\frac{2}{x}\right)^2 \times 15$
Coeff = $\frac{15}{4}$ or 3.75

Q10
(i) $(2+3x)^5 = 32 + 240x + 720x^2$

(ii) $(1+ax)(2+3x)^5$
$\rightarrow (1 \times 720) + (a \times 240) = 0$
$\rightarrow a = -3$

Q11:
(i) $(2-x)^6$
$64 - 192x + 240x^2$

(ii) $(1+2x+ax^2)(2-x)^6$
Coeff of $x^2 = 240 - 384 + 64a$
Equates to 48
$\rightarrow a = 3$

Q12 $(x+k)^8$
(i) $k^8 + 8k^7x + 28k^6x^2 + 56k^5x^3$

(ii) $28k^6 = 56k^5$
$\rightarrow k = 2$

Q13 $\left(2x-\frac{3}{x}\right)^5$
(i) $32x^5 - 240x^3 + 720x$

(ii) $\left(1+\frac{2}{x^2}\right)(32x^5 - 240x^3 + 720x)$
Coeff of x $(1 \times 720) + (2 \times -240)$
→ 240

Q14
(i) $1 + 5ax + 10a^2x^2$

(ii) $\times(1-2x) \rightarrow 5ax - 2x$
$\rightarrow a = \frac{2}{5}$

(iii) Coeff of x^2 is $-10a + 10a^2$
$\rightarrow -4 + 1.6 = -2.4$

Q15:
(i) $\left(x-\frac{2}{x}\right)^6 = x^6 - 12x^4 + 60x^2$

(ii) $\times(1+x^2) \rightarrow 60 - 12 = 48$

Q16
$(1+ax)^6$
Term in $x = 6ax$
Equate with $-30 \rightarrow a = -5$

Term in $x^3 = \frac{6.5.4}{3!}a^3$

→ coefficient of -2500

Q17
(i) $1 + 8(-2x^2) + {}^8C_2(-2x^2)^2$
$1 - 16x^2 + 112x^4$

(ii) $(2-x^2) \times$ *their* $(1 - 16x^2 + 112x^4)$
$(2 \times$ *their* $112) -$ *their* (-16)
240

Q18
${}^9C_6$ or ${}^9C_3$ used
$\left(\frac{1}{x^2}\right)^3$ seen
-84

Q19:
${}^7C_2x^5\left(\frac{2}{x^2}\right)^2$ SOI and leading to final answer

84 or $84x$ as final answer

Q20
$(1-\frac{3}{2}x)^6$
(i) Term in x^2 ${}^6C_2 \times \left(\frac{\pm 3x}{2}\right)^2 = \frac{135x^2}{4}$
Term in x^3 ${}^6C_3 \times \left(\frac{\pm 3x}{2}\right)^3 = \frac{-540x^3}{8}$

(ii) Term in $x^3 = \frac{270x^3}{4} - \frac{135kx^3}{2}$
→ $k = 1$.

Q21
(a) $ar^2 = 20$
$\frac{a}{1-r} = 3a$
Soln of equations → $(r = \frac{2}{3})$ $a = 45$

(b) $a + 7d = 3(a + 2d)$
$\rightarrow 2a = d$
$S_8 = 4(2a + 7d) = 32d$ or $64a$
$S_4 = 2(2a + 3d) = 8d$ or $16a$

Q22
$6C4 \times [2(x)]^4 \times \left[\frac{1}{(x^2)}\right]^2$
240

Q23:
(i) $(2-y)^5 = 32 - 80y + 80y^2$

(ii) $(2-(2x-x^2))^5$ "y" = "$2x - x^2$"
→ 80 + 320 = 400

Q24
$k^2 \times \left(\frac{1}{3(x)}\right)^2 \times 10$ (or correct factorials)

$10 \times k^2 \times \frac{1}{9} = 30 \Rightarrow k = 3$